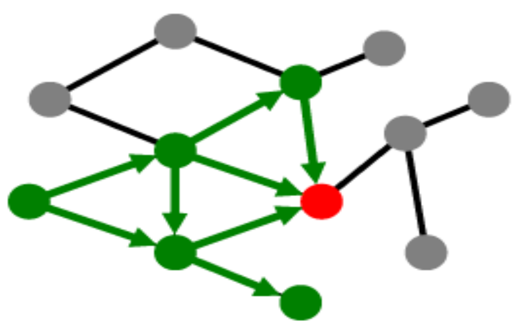
How to model diffusion?

- **Probabilistic models:**
  - Models of influence or disease spreading
    - An infected node tries to "push" the contagion to an uninfected node
  - **Example:**
    - You "catch" a disease with some prob. from each active neighbor in the network



- **Decision based models:**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if *k* of your friends do so too

Decision-based diffusion models

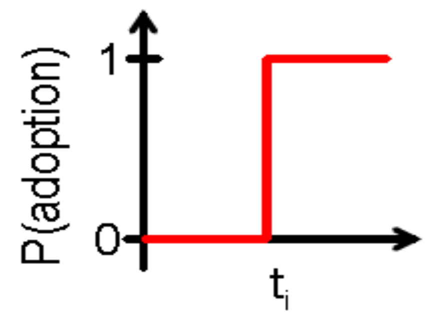
Collective action

[Granovetter 1978]

- **Collective Action** [Granovetter, ‘78]
  - Model where everyone sees everyone else’s behavior
  - **Examples:**
    - Clapping or getting up and leaving in a theater
    - Keeping your money or not in a stock market
    - Neighborhoods in cities changing ethnic composition
    - Riots, protests, strikes

The model of collective action

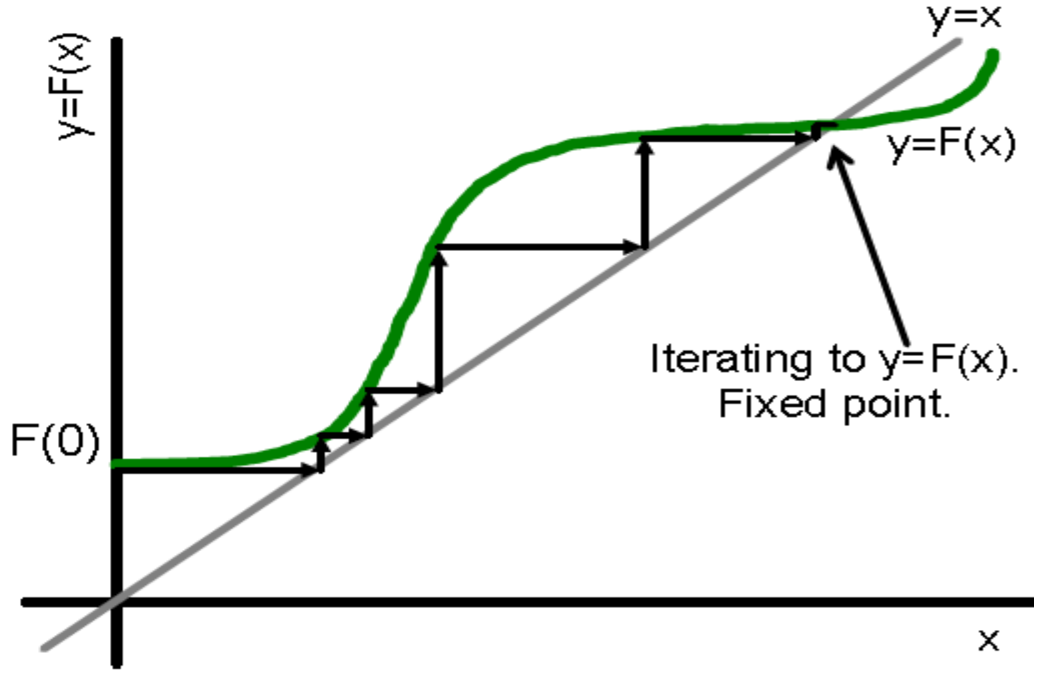
- **n people – everyone observes all actions**
- Each person *i* has a threshold t_i
  - Node *i* will adopt the behavior iff at least t_i other people are adopters:
    - **Small t_i:** early adopter
    - **Large t_i:** late adopter



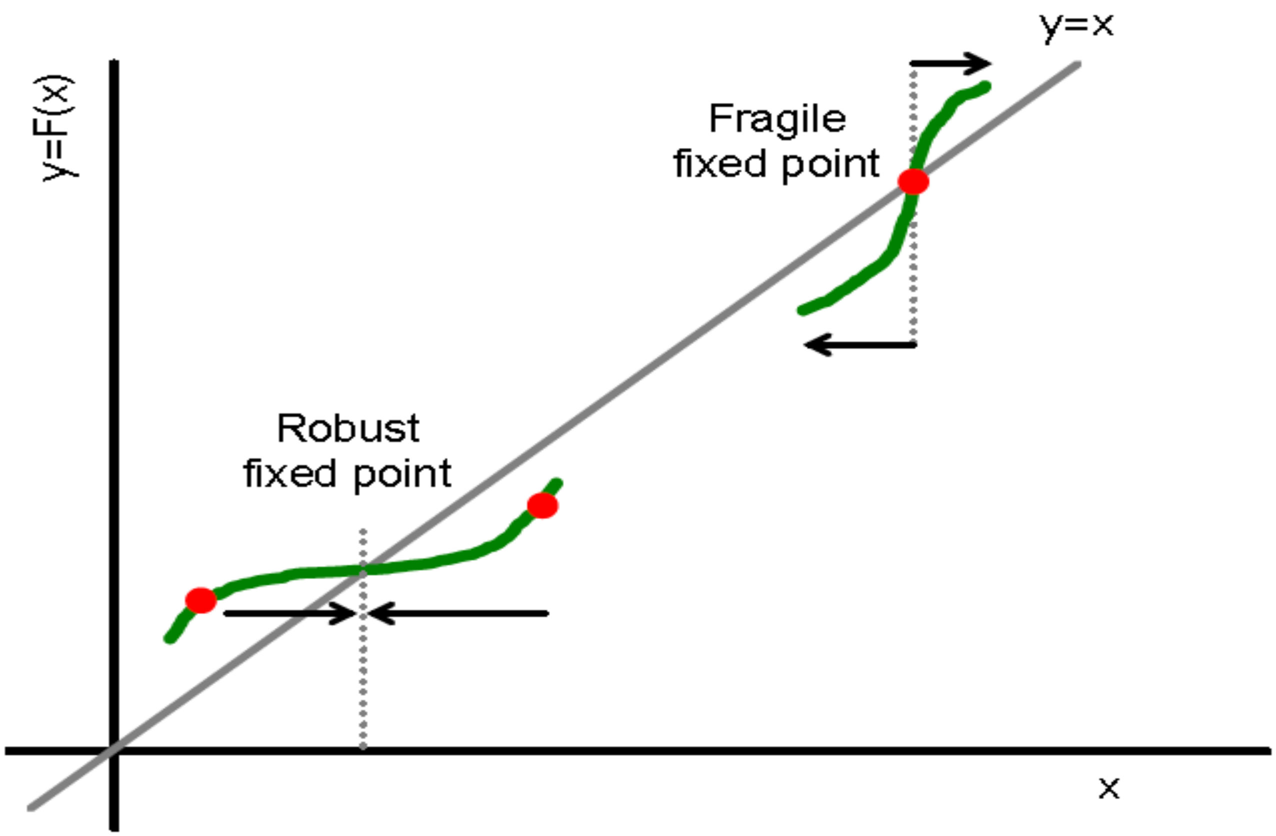
- **The population is described by {t_1,…,t_n}**
  - **F(x)** … fraction of people with threshold $t_i \leq x$

Dynamics of collective action

- **Think of the step-by-step change in number of people adopting the behavior:**
  - **F(x)** … fraction of people with threshold $\leq$ x
  - **s(t)** … number of participants at time t
- **Easy to simulate:**
  - $s(0) = 0$
  - $s(1) = F(0)$
  - $s(2) = F(s(1)) = F(F(0))$
  - $s(t+1) = F(s(t)) = F^{t+1}(0)$
- **Fixed point: F(x)=x**
  - There could be other fixed points but starting from 0 we never reach them



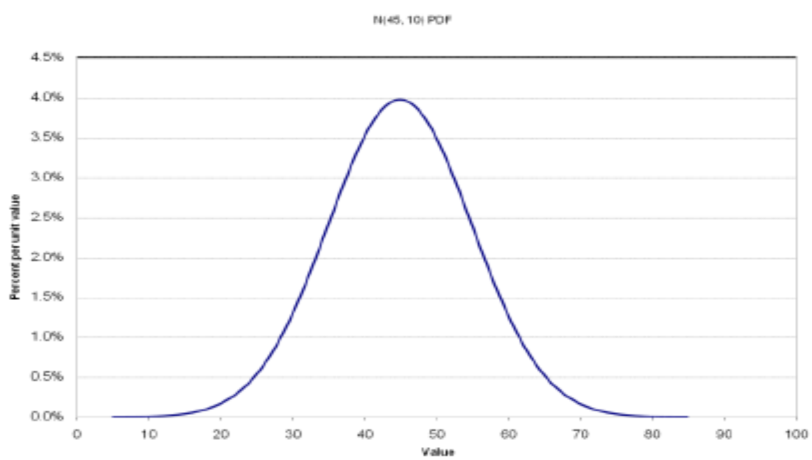
Fragile vs. robust fixed points



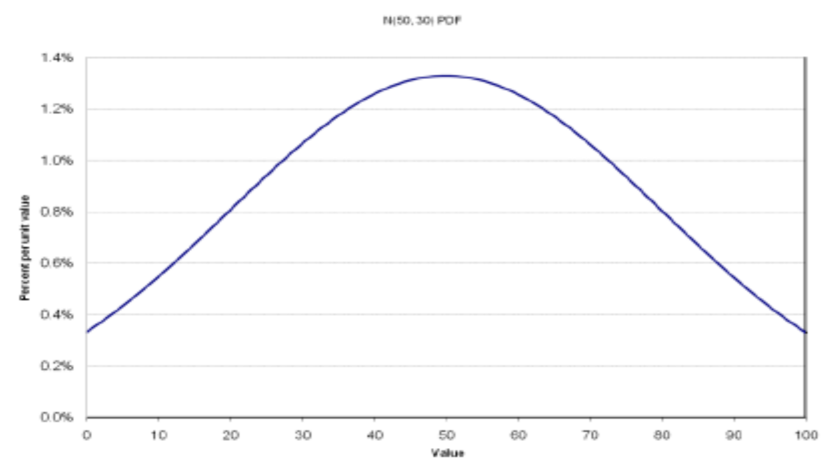
Distribution of thresholds (trust)

- Each threshold t_i is drawn independently from some distribution $F(x) = Pr[thresh \leq x]$
  - **Suppose:** Normal with μ=n/2, variance σ

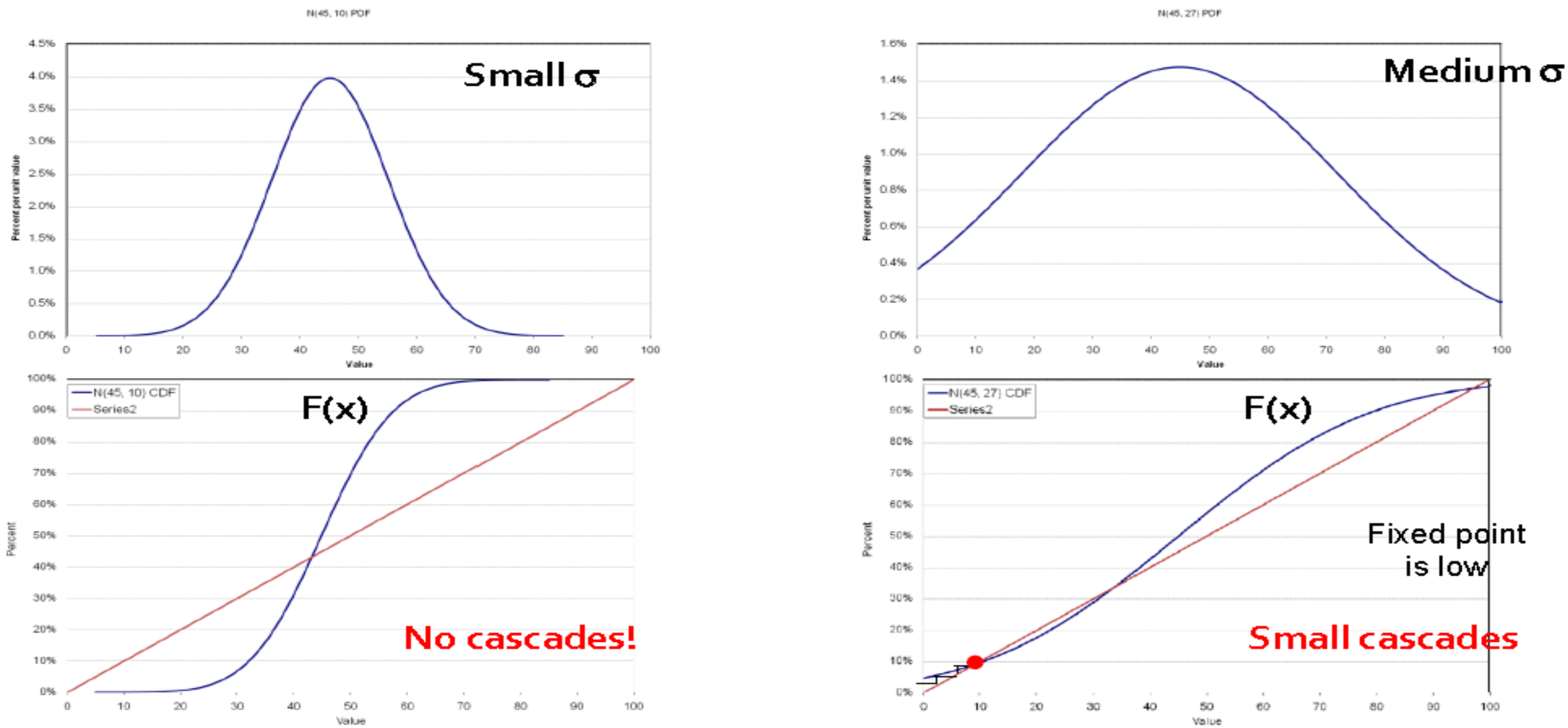
Small σ:



Large σ:

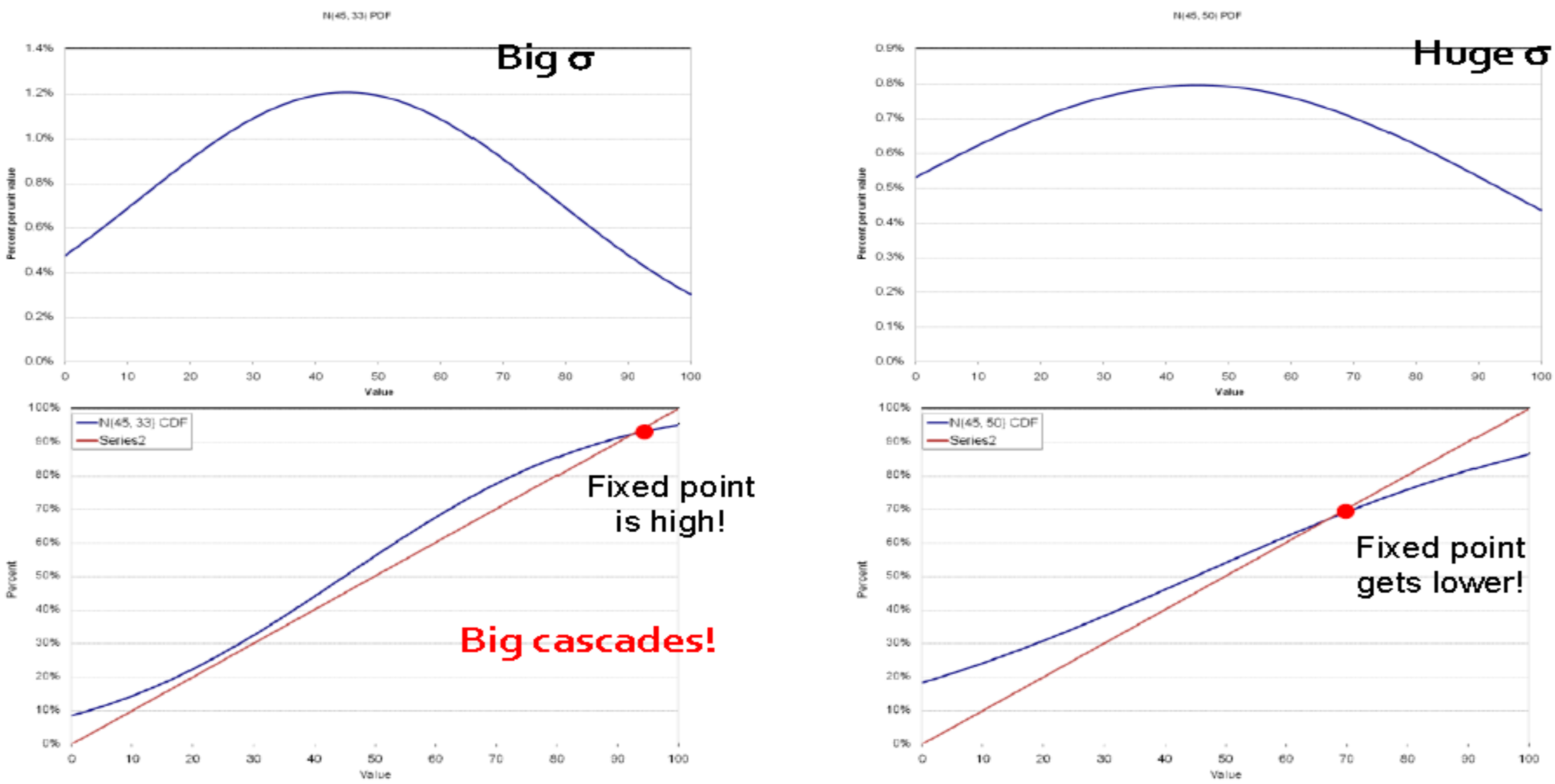


Simulation

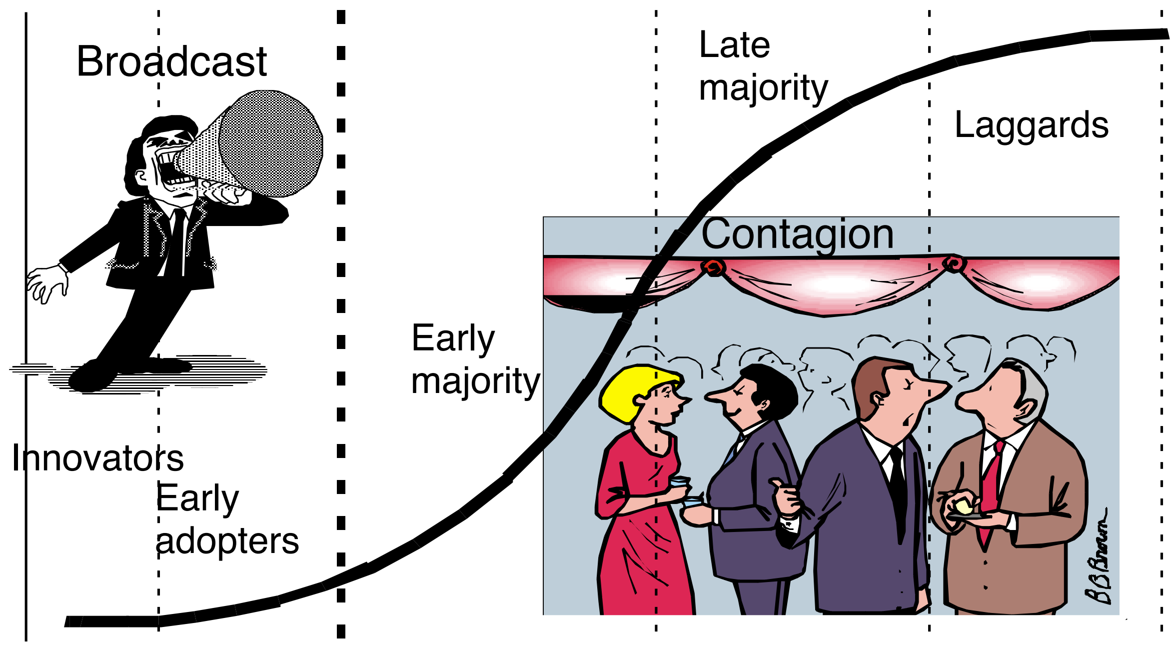


Bigger variance let's you build a bridge from early adopters to mainstream

Simulation



But if we increase the variance even more we move the higher fixed point lover


Broadcast
Late
majority
Laggards
Contagion
Early
majority
Innovators
Early
adopters

Weaknesses of the CA model

- **It does not take into account:**
  - No notion of social network – more influential users
  - It matters who the early adopters are, not just how many
  - Models people's awareness of size of participation not just actual number of people participating
  - **Modeling thresholds**
    - Richer distributions
    - Deriving thresholds from more basic assumptions
      - game theoretic models

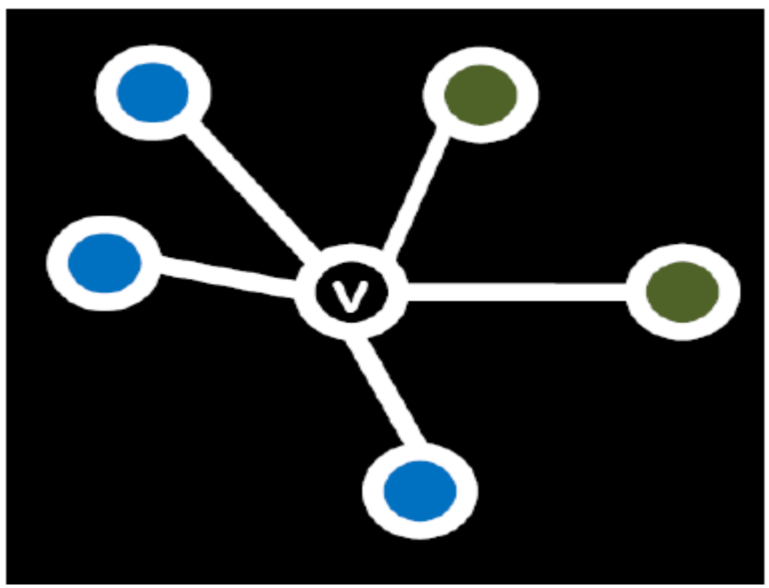
Decision-based diffusion models

Game-theoretic models of cascades

[Moore 2000]

Game theoretic models of cascades

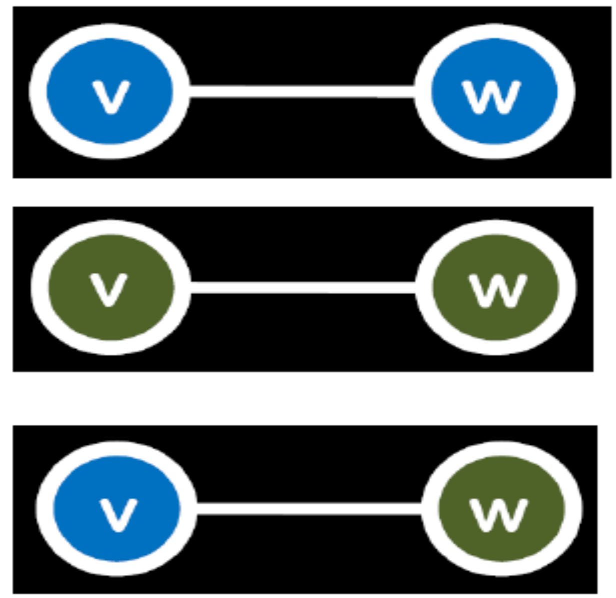
- Based on 2 player coordination game
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** "behavior", **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node v

Rules of the game

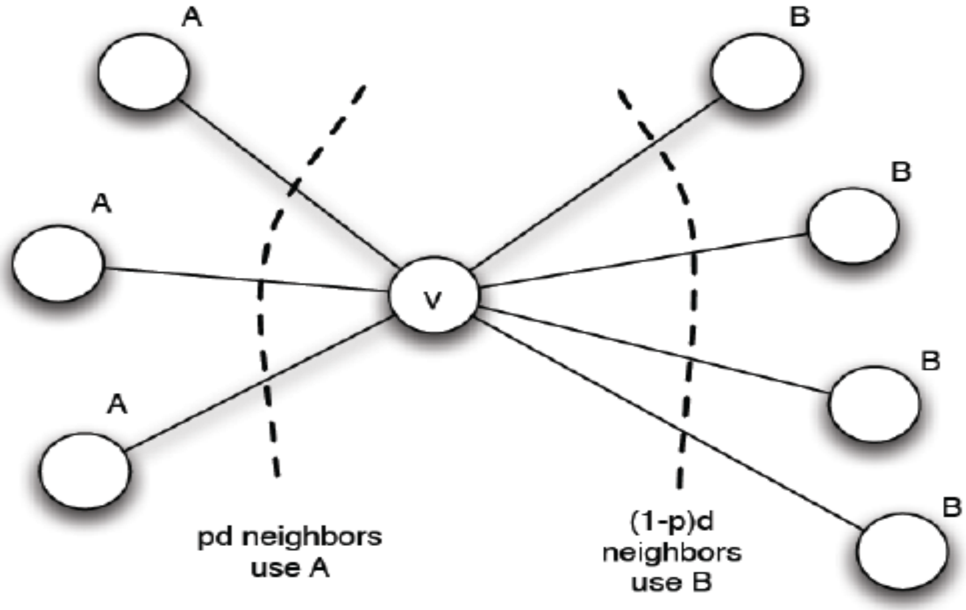
- ***Payoff matrix:***
  - If both v and w adopt behavior A, they each get payoff *a>0*
  - If *v* and *w* adopt behavior *B*, they reach get payoff *b>0*
  - If *v* and *w* adopt the opposite behaviors, they each get *0*



- In some large network:
  - Each node *v* is playing a copy of the game with each of its neighbors
  - **Payoff**: sum of node payoffs per game

v \ w	A	B
A	a, a	$0, 0$
B	$0, 0$	b, b

Decision rule for node *v*



Threshold:
v choses *A* if $p>q$

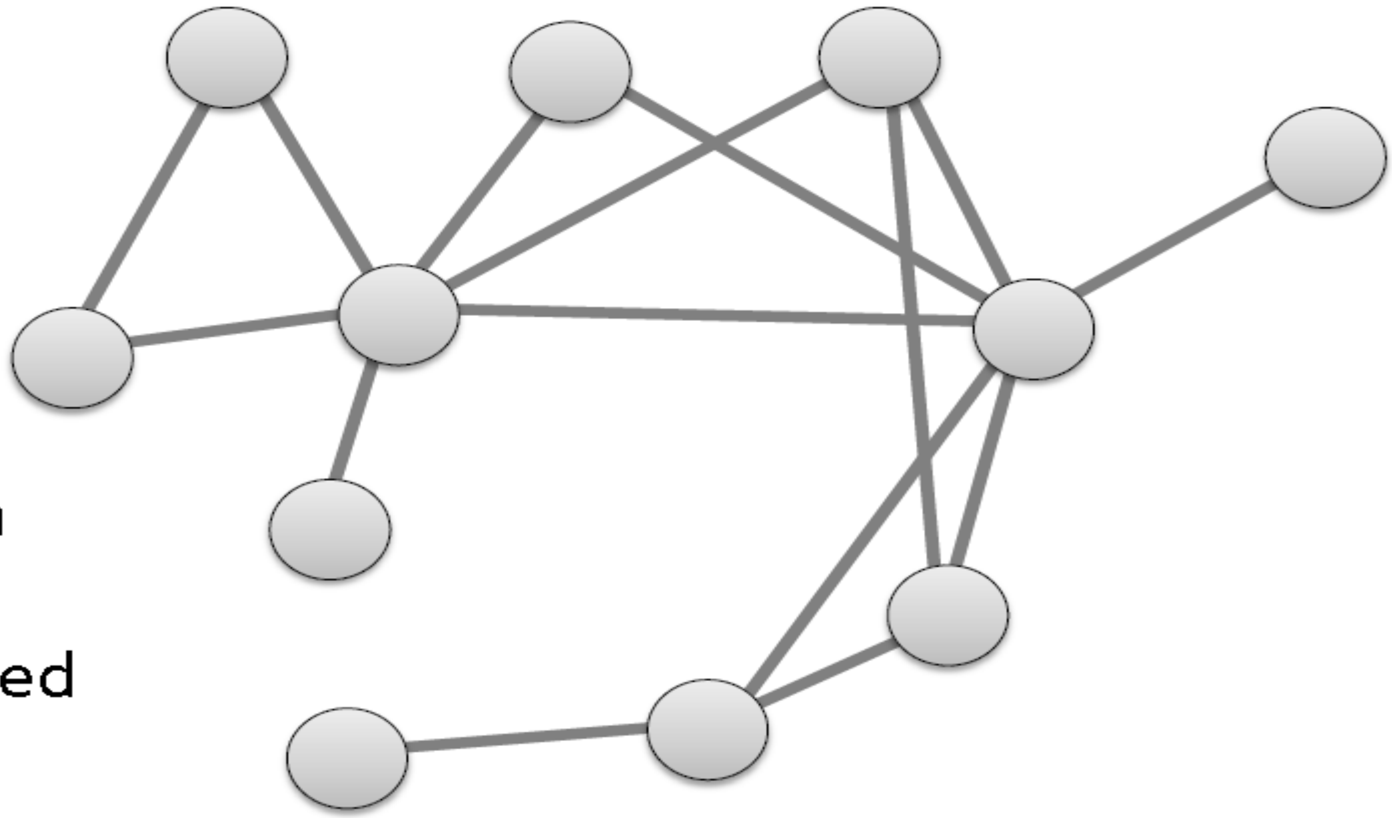
$$q = \frac{b}{a+b}$$

- Let v have *d* neighbors
- Assume fraction *p* of *v*'s neighbors adopt *A*
  - $Payoff_v = a \cdot p \cdot d$ if *v* chooses A
    $= b \cdot (1-p) \cdot d$ if *v* chooses B
- **Thus: *v* chooses A if:** $a \cdot p \cdot d > b \cdot (1-p) \cdot d$

Example

- **Scenario:**
  Graph where everyone starts with B.
  Small set S of early adopters of A
  - Hard wire S – they keep using A no matter what payoffs tell them to do
- Payoffs are set in such a way that nodes say:
  If at least 50% of my friends are red I'll be red
  (this means: $a = b+\varepsilon$)

$S = \{u, v\}$



If more than 50% of my friends are red I'll be red

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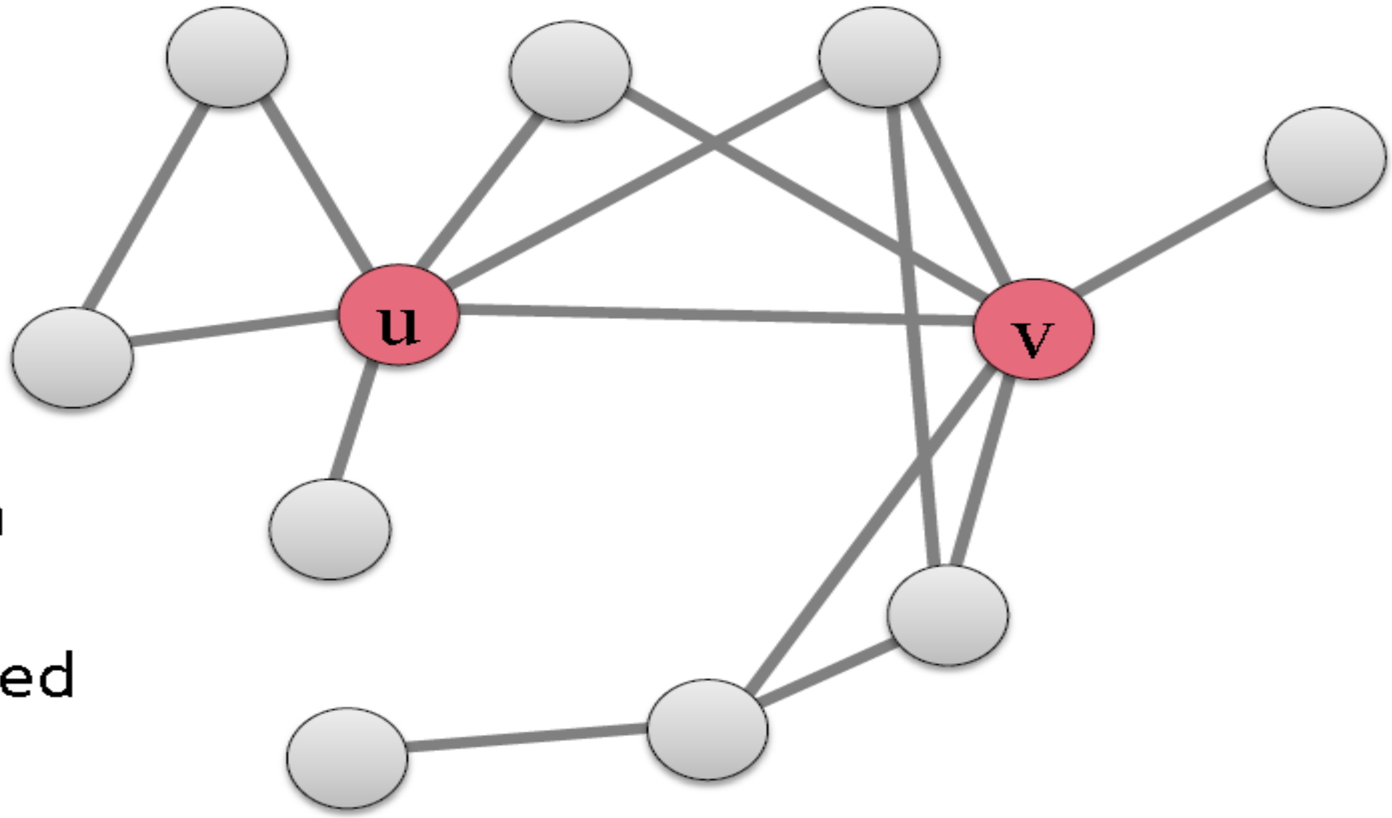

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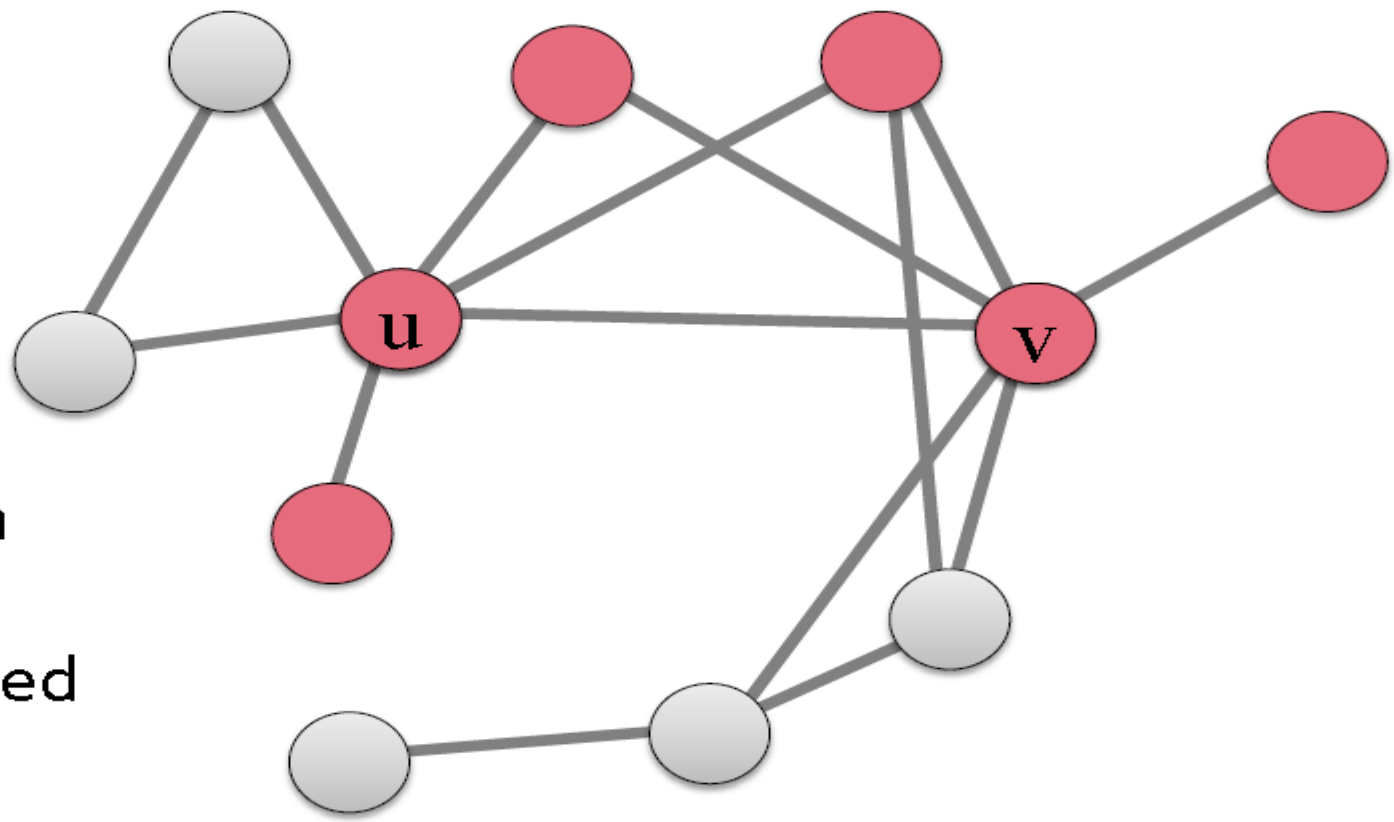

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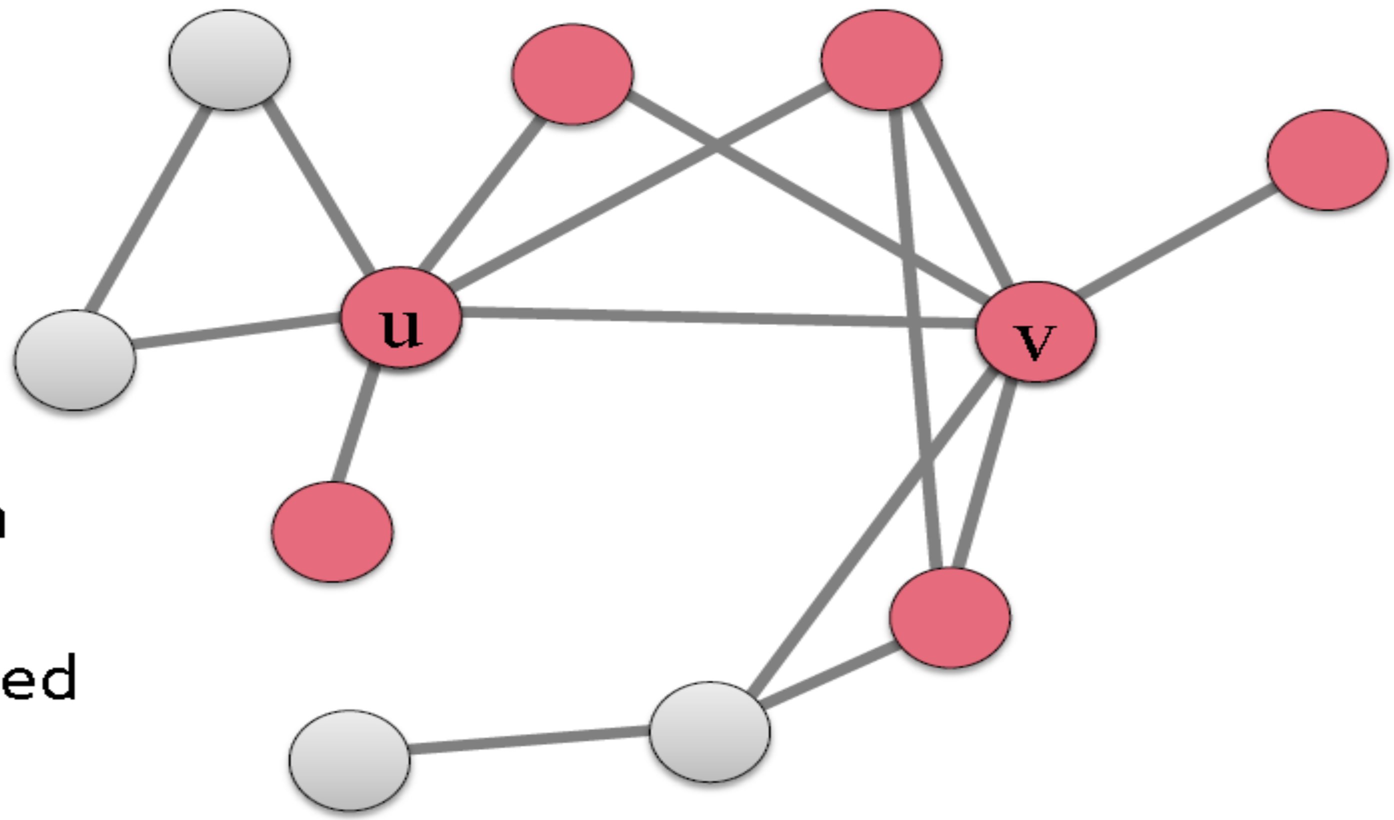

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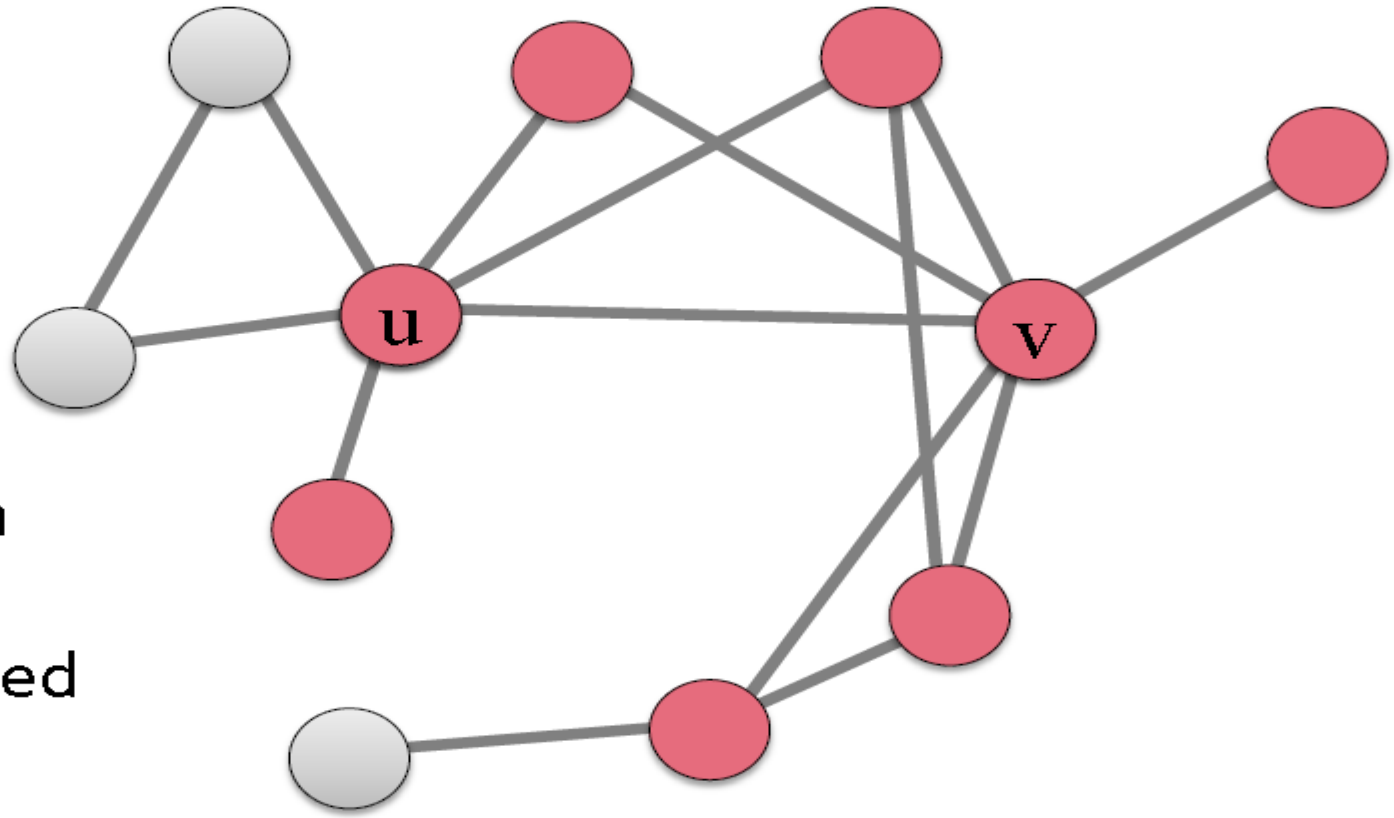

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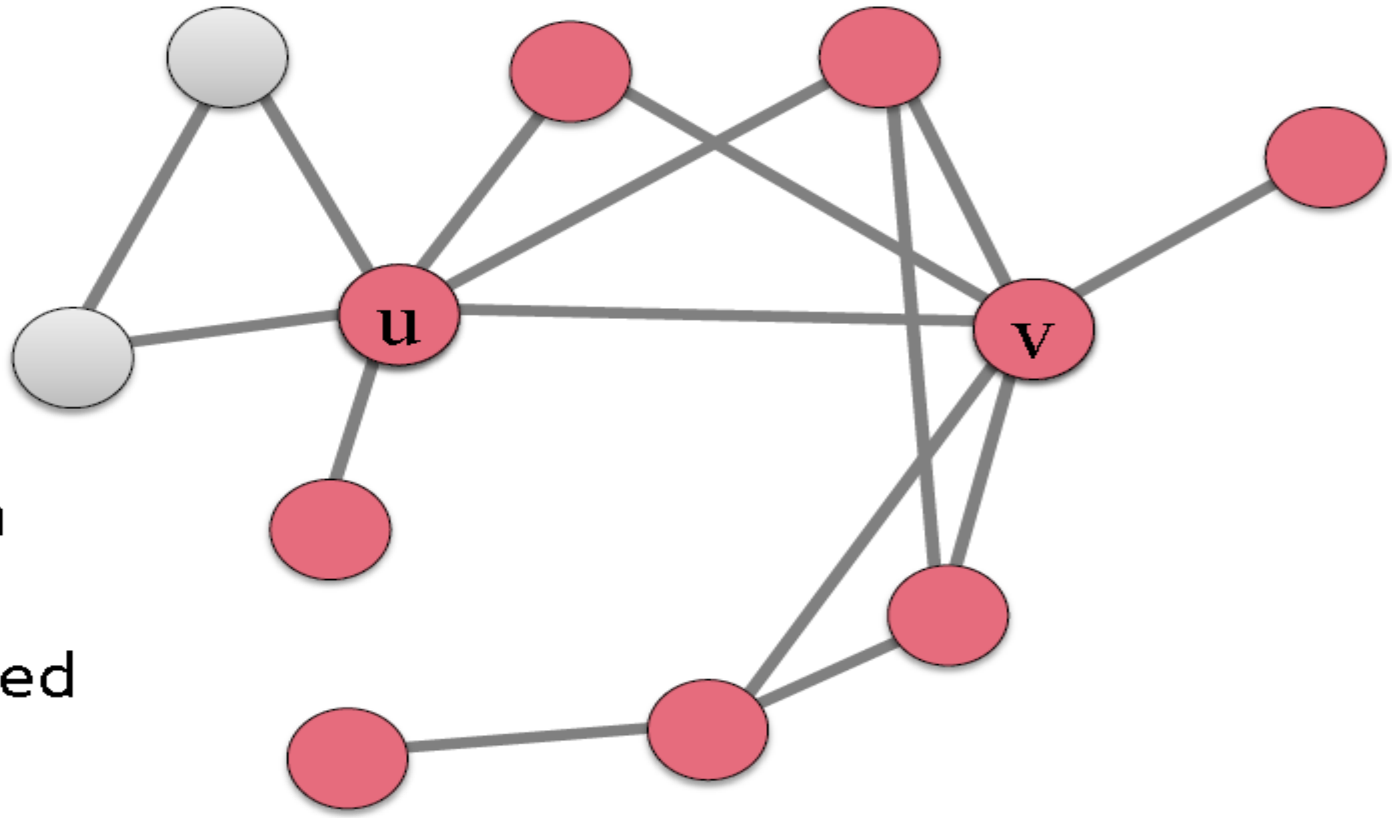
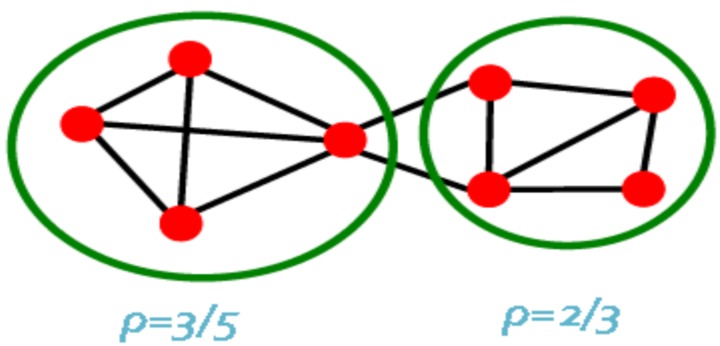


If more than 50% of my friends are red I'll be red

Stopping cascades

- **What prevents cascades from spreading?**
- Def: **Cluster of density ρ** is a **set of nodes C** where each node in the set has at least ρ fraction of edges in C.



Stopping cascades

- Let S be an initial set of adopters of A
- All nodes apply threshold *q* to decide whether to switch to A
- **Two facts:**
  - 1) If *G\S* contains a cluster of density *>(1-q)* then *S* can not cause a cascade
  - 2) If S fails to create a cascade, then there is a cluster of density >(1-q) in G\S