Dynamic pricing

Part 1

The problem

We have an e-commerce site.

Each time an user enters the site, coming form a click on an advertisement, we offer him the same product.

The key point is that we are able to offer a different price each time.

We do not know which is the *best* price, i.e. the price which gives us the maximum revenue.

The basic equation is

$$R = p \times d(p)$$

where

R is the total revenue, given by the price p multiplied by the demand d.

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The demand is a function of the price *d(p)*. This means that for different prices the demand will be different. Usually (not always) the greater is the price the lower is the demand at that price level.

You can think of demand as the number of customers which accept buying our product at the price we offer.

Note these assumptions:

1. We have a single product.
2. Each customer is exposed at a certain price and decides whether to buy or not, i.e. she can buy 0 or 1 unit, not more than one.

For the second assumption, the demand *d(p)* is the fraction of population available to buy at price *p*.

Pricing as MAB problem

Let us interpret the pricing problem as a Multi Armed Bandit problem.

The framework used for advertisements and clickthrough rate is still useful, with some adjustment to fit with new needs.

Prices play the role of advertisements.

Each price is shown to the customer a certain amount of times and is evaluated on the revenue generated.

Then we apply some policy with exploration and exploitation.

The quality of a price is a real number, not a probability.

Price	Offers	Accepted	Selling rate	Revenue	Expected revenue
100€	200	18	9%	1,800€	9.00€
125€	150	12	8%	1,500€	10.00€
150€	100	6	6%	900€	9.00€

The quality of each price is its expected revenue.

Applying Epsilon-Greedy is easy: the leader is the price 125€.

If ε = 15%, then we select

1. price 100€ with probability 5% (i.e. 15% : 3)
2. price 125€ with probability 90% (i.e. 1 – 15% + 15% : 3)
3. price 150€ with probability 5% (i.e. 15% : 3)

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Applying Softmax is easy, too.

In the formula

$$p_i = \frac{e^{-\frac{Q_i}{T}}}{\sum_j e^{-\frac{Q_j}{T}}}$$

the value Q_1 is 9, Q_2 is 10, Q_3 is 9.

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Applying Thompson Sampling means using the selling rate as it was a CTR.

For example, price 100€ will be associated with a Beta(19, 183) (remember: you have to add 1 to hits and failures counts).

At each round we can sample the Beta distribution for each price, get a selling rate, multiply it for the price itself, use the outcome as expected value and select the price with the greatest expected value.

If sampling Beta(19, 183) we get 0.092 (sampled selling rate), then we multiply 100 x 0.092 and obtain 9,2€ as expected revenue for this round.