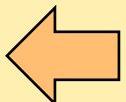


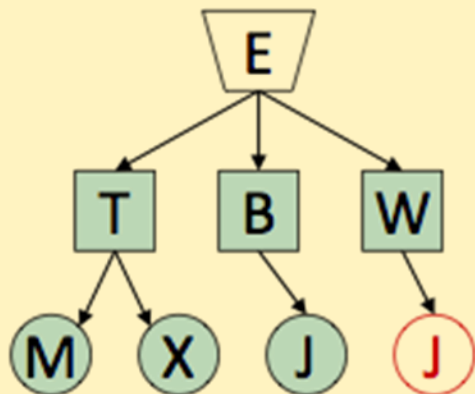
- A **symbolic model** is a subjective formal representation of ideas and knowledge about some aspects of the real world (*domain of discourse*), designed to serve an explicit purpose.
- A **data model** is a set of abstraction mechanisms to describe abstract knowledge
  - Conceptual data model: to analyse a problem, given user requirements
    - Operational databases:
      - E.g., E-R or Entity-Relationship, **ODM or Object Data Model**
    - Data warehouses:
      - **Dimensional Fact Model**
  - Logical model: to design a solution independently of actual DBMS
    - E.g., Relational Data Model  **Today: RECALLS ON RELATIONAL MODEL**
  - Physical model: to realize a project on a specific DBMS

What is the problem?

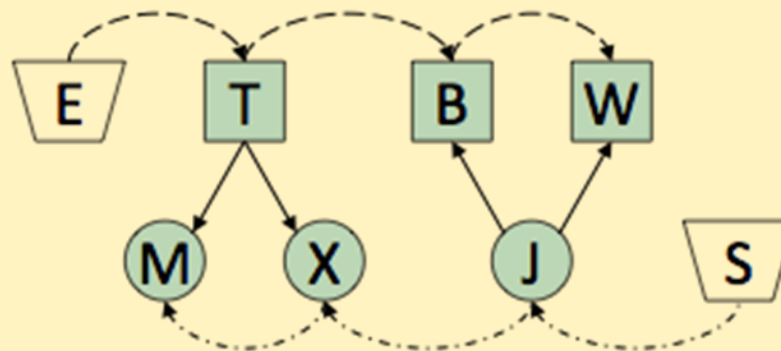
How to solve it?

How to implement a solution?

## Hierarchical (tree)



## Network (graph)



## Relational (table)

E	
B	...
T	...
W	...

S	
J	...
M	...
X	...

R	
M	T
X	T
J	B
J	W

The relational data model is based on typed finite sets of ...

**Simplicity is Beautiful**

## Definition.

- integers, floats, booleans, and strings are **primitive** types;
- if  $T_1, \dots, T_n$  are primitive types, and  $A_1, \dots, A_n$  are distinct **attribute names**, then  $(A_1: T_1, \dots, A_n: T_n)$  is a **tuple type** of degree  $n$ . The **attributes order is unimportant**;
- A tuple  $(A_1 := V_1, \dots, A_n := V_n)$  of type  $T = (A_1: T_1, \dots, A_n: T_n)$  is a **set of pairs**  $(A_i, V_i)$  with  $V_i$  of primitive type  $T_i$ .

**(Age :int, Name :string)**

**(Age := 23, Name := 'mary')**

**= { (Age, 23), (Name, 'mary') }**

**= { (Name, 'mary'), (Age, 23) } =**

**(Name :string, Age :int)**

**(Name := 'mary', Age := 23)**

## Definition.

- integers, floats, booleans, and strings are **primitive** types;
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- A tuple  $(A_1 := V_1, \dots, A_n := V_n)$  of type  $T = (A_1: T_1, \dots, A_n: T_n)$  is a **set of pairs**  $(A_i, V_i)$  with  $V_i$  of primitive type  $T_i$ .

## When two tuple types are equal?

Two **tuple types** are **equal** iff they have the same degree, same attributes and same type of the attributes with the same name.

## When two tuples are equal?

Two **tuples** of the **same type** are **equal** iff they have the same set of pairs (Attribute, Value)

**Definition.** A relational database is described by a set of **relation schemes**  $R:\{T\}$  defined as follows:

- $T = (A1: T1, \dots, An: Tn)$  is a **tuple type**;
- $\{T\}$  is called a **relation type**;
- a **relation scheme**  $R:\{T\}$  is a variable  $R$  with a relation type  $T$ ;

For brevity, instead of  $R:\{T\}$  we will write  $R(T)$ .

In the following we will use as schema notation  $R(A1, \dots, An)$  when the type of the attributes is not important.

**Student(Age :int, Name :string)**

**Student(Name :string, Age :int)**

**When two relation types are equal?**

**Two relation types are equal iff they have the same tuple types.**

**Definition.** Consider a relation scheme  $R:\{T\}$ :

- A **schema instance** (or **relation**) is a finite set of tuples with type  $T$ .

**Student**(Age :int, Name :string)

{(Age := 23, Name := 'mary' ),  
(Age := 24, Name := 'john' ) }

{(Name := 'john, Age := 24),  
(Age := 23, Name := 'mary' ) }

**Important properties of relations:** finite sets of tuples without duplicates,  
The **order** of a relation elements and of the attributes is **irrelevant**.

**When two relations are equal?**

Two **relations** of the **same type** are **equal** iff have the equal tuples.

**In some books on relational algebra,  
and in relational DBMS (SQL)  
the order of the attributes is IMPORTANT**

**Two relations  $R$  and  $S$  have the same type if**

- they have the same set of attributes,**
- the order of the attributes is the same for both relations.**

**Relational schema:**  $R(A : \text{int}, B : \text{string})$

An instance of  $R$  is a finite set of tuples with type  $(A : \text{int}, B : \text{string})$

$R = \{ (A := 10, B := \text{"Mario"}),$   
 $(A := 18, B := \text{"Giovanna"}),$   
 $(A := 77, B := \text{"Luca"}),$   
 $(A := 18, B := \text{"Francesco"}),$   
 $(A := 14, B := \text{"Carla"}) \}$

**R**

A	B
10	Mario
18	Giovanna
77	Luca
18	Francesco
14	Carla



**Key:** A key for a relation is a minimal subset of attributes whose values identify a tuple;

**Primary key:** one of the possible keys, usually the shortest.

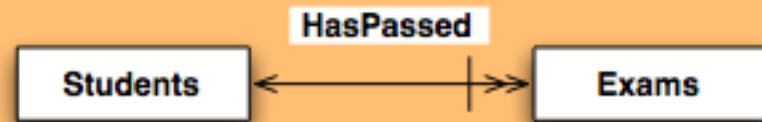
Which are the keys?

**Students**(Name: string, StudCode: string, City: string, BirthYear:int)

**Exams**(Subject: string, Candidate: string, Date: string, Grade: int)

Relationships between tuples are represented with values of a set of attributes (**foreign key**) which take as values those of the primary key of another relation.

# EXAMPLE



## Schema:

Students(Name: string, StudCode: string, City: string, BirthYear:int)

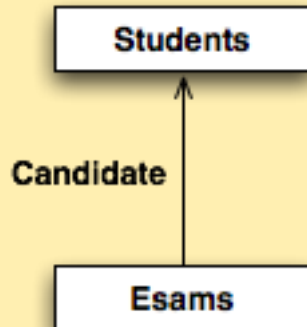
Exams(Subject: string, Candidate\*: string, Date: string, Grade: int)

### Students

Name	<u>StudCode</u>	City	BirthYear
Isaia	171523	PI	1982
Rossi	167459	LU	1980
Bianchi	179856	LI	1981
Bonini	175649	PI	1982

### Exams

<u>Subject</u>	<u>Candidate</u>	Grade	Date
BD	171523	20	12/01/05
ALG	167459	30	15/09/05
MP	171523	30	25/10/05
IS	167459	20	10/10/05



# EXAMPLE ALGEBRA: INTEGER ARITHMETIC

**DOMAIN:** integers

**OPERATORS:**  $-$ ,  $+$ ,  $*$ ,  $\dots$

**EXPRESSIONS:**  $e + ((2 * a) + ((c + (-d)) * 5))$

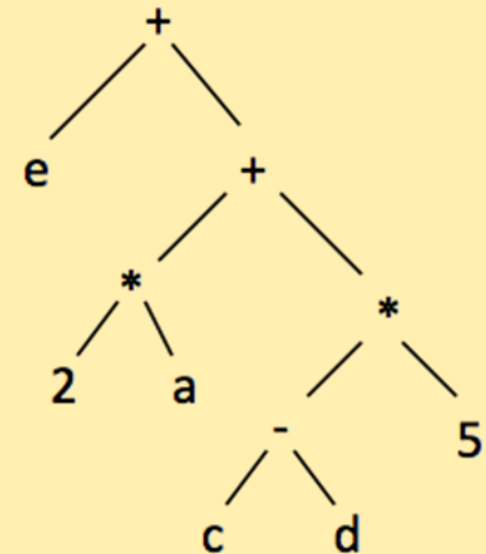
**LAWS:** Commutative, Associative, Distributive

$$\square a * b = b * a$$

$$\square a * (b * c) = (a * b) * c$$

$$\square a * (b + c) = a * b + a * c$$

**Tree form**



**Relational algebra** is a set of operators on relations whose results are themselves relations. An **expression** is called a **query**.

**Relational Algebra** describes **HOW** to get results with “**logical query plan**” of relational operators.

A naive way to evaluate an expression would be to compute the results of the relational operators directly as specified... but there are equivalence rules for **query rewriting**.

The language is not used by DBMS to query but to represent queries internally.

The **standard language** used by DBMS to query a relational database is **SQL**.

**SQL** is a **declarative** rather than a **procedural language**.

It describes **WHAT** we are looking for, but not how to get it.

**Note:** R through W stand for any expressions, not necessarily for stored relations.

**Project ( $\pi$ ):**  $\pi_{A_1, A_2, \dots, A_n}(R)$

Which is the result type?  $\{(A_1:T_1, A_2:T_2, \dots, A_n:T_n)\}$

If R has M elements, how many elements has the result?

**Rename ( $\rho$ ):**  $\rho_{A \rightarrow B}(R)$

**Generalized Project**

$\pi_{Exp_1 \text{ AS } A_1, Exp_2 \text{ AS } A_2, \dots, Exp_n \text{ AS } A_n}(R)$

Which is the result type?

Semantics


# EXAMPLE

## Students

Find name, student code and city of students

$\pi$  Name, StudCode, City (Students)

Name	<u>StudCode</u>	City	BirthYear
Isaia	171523	PI	1982
Rossi	167459	LU	1980
Bianchi	179856	LI	1981
Bonini	175649	PI	1982

Name	StudCode	City
Isaia	171523	PI
Rossi	167459	LU
Bianchi	179856	LI
Bonini	175649	PI

Find city of students

$\pi$  city (Students) ?

Which are the query result types?

**Selection ( $\sigma$ ):** selects the tuples from a relation that satisfy a condition.

$$\sigma_{\text{Condition}}(R)$$

Condition is a propositional logic expression ( $\wedge \vee \neg$ ) over comparison predicates ( $\leq, <, =, >, \geq, \neq$ ) and arithmetic expressions ( $+, -, *, /$ )

Semantics


Which is the result type?

If R has M elements, how many elements has the result?

# EXAMPLE

## Students

Find all data of Pisa students:

$\sigma_{\text{City} = \text{'PI'}}(\text{Students})$

Name	StudCode	City	BirthYear
Isaia	171523	PI	1980
Bonini	175649	PI	1980

Name	<u>StudCode</u>	City	BirthYear
Isaia	171523	PI	1982
Rossi	167459	LU	1980
Bianchi	179856	LI	1981
Bonini	175649	PI	1982

Find name, student code and birth year of Pisa students:

$\pi_{\text{Name, StudCode, BirthYear}}(\sigma_{\text{city} = \text{'PI'}}(\text{Students}))$

Name	StudCode	BirthYear
Isaia	171523	1980
Bonini	175649	1980

Which are the query result types?



Students

Name	<u>StudCode</u>	City	BirthYear
Isaia	171523	PI	1982
Rossi	167459	LU	1980
Bianchi	179856	LI	1981
Bonini	175649	PI	1982

Find the name of Pisa students:

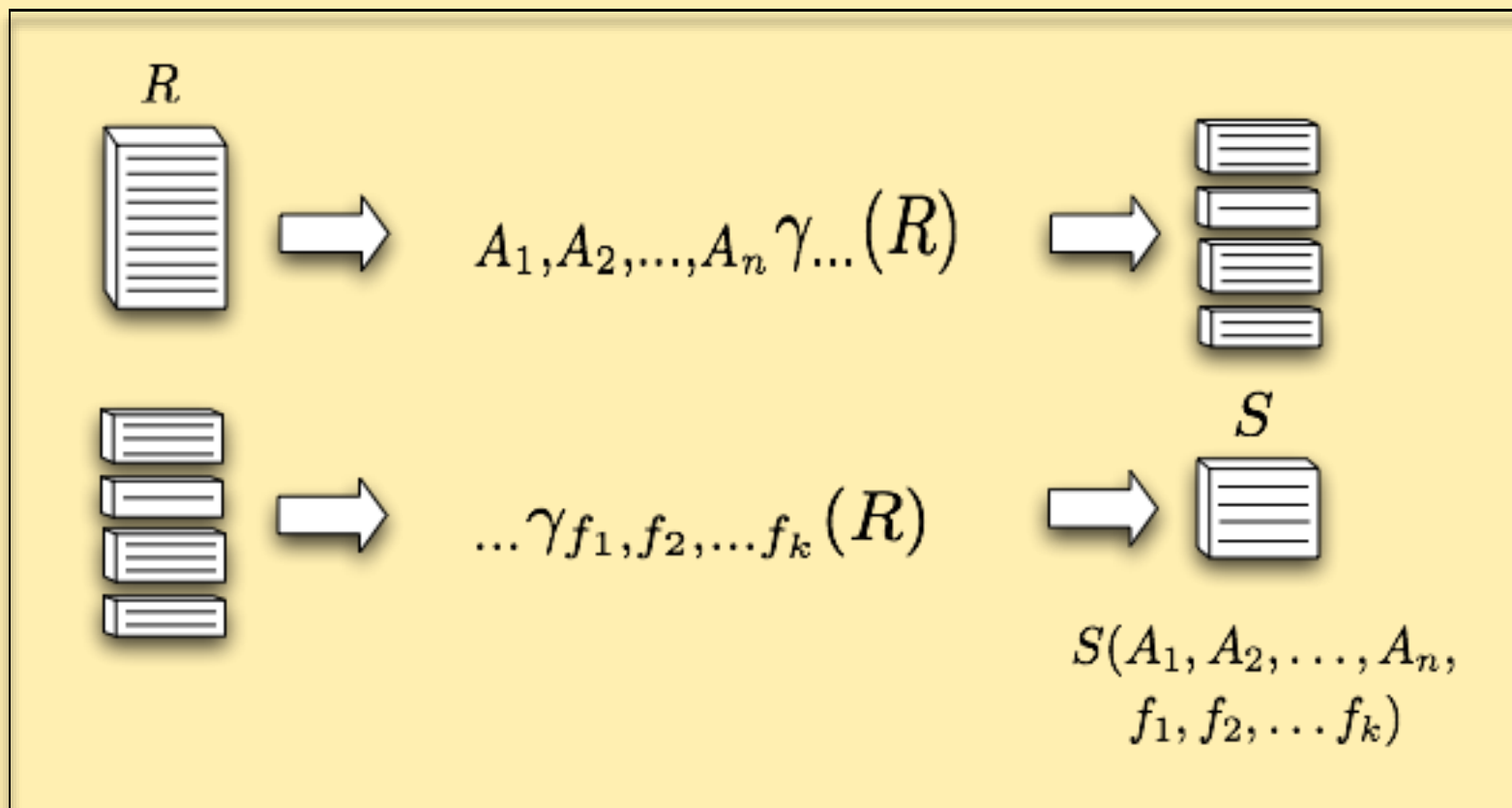
$\pi$  Name ( $\sigma$  City = 'PI' (Students))

is equivalent to:  $\sigma$  City = 'PI' ( $\pi$  Name (Students)) ?

is equivalent to:  $\sigma$  City = 'PI' ( $\pi$  Name, City (Students)) ?

is equivalent to:  $\pi$  Name ( $\sigma$  City = 'PI' ( $\pi$  Name, City (Students))) ?

$$S = A_1, A_2, \dots, A_n \gamma f_1, f_2, \dots, f_k (R)$$

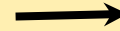


# EXAMPLE

Find for each student, the number of exams, min, max and avg grade.

Candidate  $\gamma$  COUNT(\*), MIN(Grade), MAX(Grade), AVG(Grade) (*Exams*)

<u>Subject</u>	<u>Candidate</u>	Grade	Date
BD	171523	20	12/01/18
ALG	167459	30	15/09/18
MP	171523	30	25/10/18
IS	167459	28	10/10/18



<u>Subject</u>	<u>Candidate</u>	Grade	Date
ALG	167459	30	15/09/18
IS	167459	28	10/10/18
BD	171523	20	12/01/18
MP	171523	30	25/10/18



Candidate	COUNT(*)	MIN(Grade)	MAX(Grade)	AVG(Grade)
167459	2	28	30	29.0
171523	2	20	30	25.0

Type of result is:

{(Candidate:int, COUNT(\*):int, MIN(Grade):int, MAX(Grade):int, AVG(Grade):float)}

**Grouping ( $\gamma$ ):**

$$A_1, A_2, \dots, A_n \gamma f_1, f_2, \dots, f_k (R)$$

where  $A_i$  are attributes of  $R$  and the  $f_i$  are aggregation function (**min, max, count, sum, avg, ...**)

Which is the result type?  $\{(A_1:T_1, A_2:T_2, \dots, A_n:T_n, f_1:T_{f1}, f_2:T_{f2}, \dots, f_k:T_{fk})\}$

If  $R$  has  $M$  elements, how many elements has the result?

**Generalized Grouping**

$$A_1, A_2, \dots, A_n \gamma f_1 \text{ AS } F_1, f_2 \text{ AS } F_2, \dots, f_k \text{ AS } F_k (R)$$

Which is the result type?

**Union ( $\cup$ ):** the arguments R and S **must have the same type** and the result is the union of the two set of tuples.

$$R \cup S$$

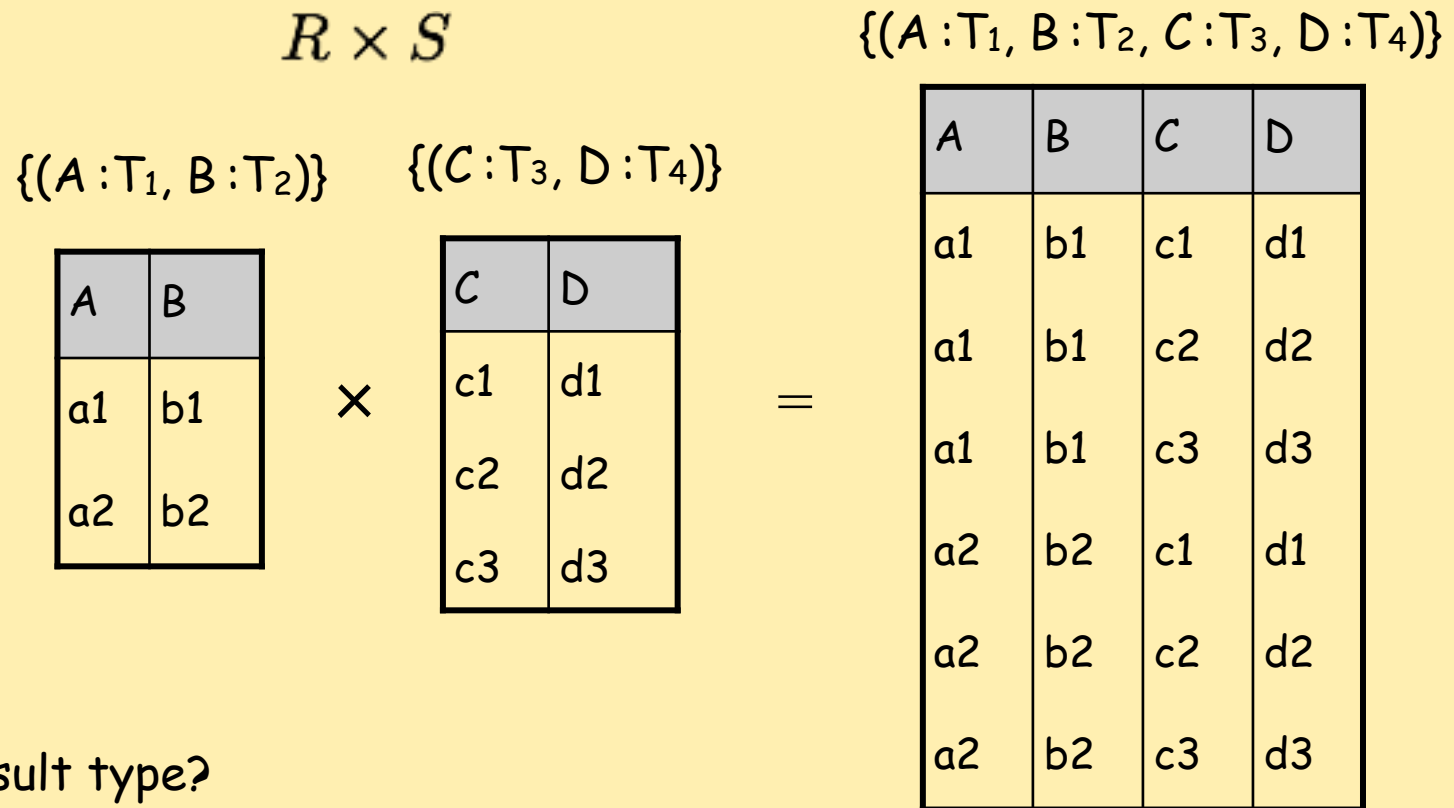
**Set-difference ( $-$ ):** the arguments R and S **must have the same type** and the result is the tuples in R but not in S .

$$R - S$$

Which is the query result type?

If R and S have  **$M_r$**  and  **$M_s$**  elements, how many elements has the result?

**Product ( $\times$ ):** of two relations  $R(A_1:T_1, \dots, A_n:T_n)$  and  $S(B_1:T_1, \dots, B_m:T_m)$  with different attributes



Which is the result type?

If  $R$  and  $S$  have  $M_r$  and  $M_s$  elements, how many elements has the result?

Students(Name, StudCode, City, BirthYear)  
 Exams(Subject, Candidate\*, Date, Grade)

Find the name of students which have passed BD with 30

$\pi_{\text{Name}}(\sigma_{\text{Subject} = \text{'BD'} \wedge \text{Grade} = 30}(\sigma_{\text{StudCode} = \text{Candidate}}(\text{Students} \times \text{Exams})))$

Is it meaningful?

Simpler with **join**!  $\sigma_{R.A = S.D} (R \times S) = R \bowtie_{R.A = S.D} S$

$\pi_{\text{Name}}(\sigma_{\text{Subject} = \text{'BD'} \wedge \text{Grade} = 30}(\text{Students} \bowtie_{\text{StudCode} = \text{Candidate}} \text{Exams}))$

**Intersect**

$$R \cap S$$

**Natural Join**

$$R \bowtie S$$



Examples for the relations  $R(A, B, C, D)$ ,  $S(E, F)$ , and  $T(G, H)$  :

$$\begin{aligned}\pi_A(\pi_{A,B}(R)) &\equiv \pi_A(R) \\ \sigma_{C_1}(\sigma_{C_2}(R)) &\equiv \sigma_{C_1 \wedge C_2}(R) \\ \sigma_{C_R \wedge C_S}(R \bowtie S) &\equiv \sigma_{C_R}(R) \bowtie \sigma_{C_S}(S) \\ R \bowtie (S \bowtie T) &\equiv (R \bowtie S) \bowtie T \\ (R \bowtie S) &\equiv (S \bowtie R) \\ \sigma_{C_X}(X \gamma_F(R)) &\equiv X \gamma_F(\sigma_{C_X}(R))\end{aligned}$$