Logistics examples

1) Nurse Rostering

- W : set of workers
- 7 : available weekly shifts
- m_j : minimum number of workers required on day j, $j = 1, \dots 7$

m_1	m_2	m_3	m_4	m_5	m_6	m_7
18	27	22	26	25	21	19

- w_i : wage (per worker) related to shift i, $i = 1, \dots 7$

w_1	w_2	w_3	w_4	w_5	w_6	w_7
680	705	705	705	705	680	655

The problem : To schedule workers for each available shift (nurse rostering) so as to satisfy the minimum worker requirement per day at a minimum cost

I.L.P. model

$x_i \geq 0$, integer : workers scheduled for shift i, $i = 1, .. 7$

$$\text{Min} \sum_{i=1}^{7} w_i \cdot x_i$$

$$\sum_{i=1}^{7} a_{ij} x_i \geq m_j \qquad j = 1, .. 7$$

$$x_i \geq 0 \text{, integer} \qquad i = 1, .. 7$$

where coefficients a_{ij} give day on / day off information for each shift:

$$a_{ij} = \begin{cases} 1 & \text{if a worker works on day } j \text{ according to shift } i \\ 0 & \text{otherwise} \end{cases} \qquad \begin{array}{l} i = 1, .. 7 \text{ (shifts)} \\ j = 1, .. 7 \text{ (days)} \end{array}$$

The optimal solution:

$x_1^* = 4 \quad x_2^* = 0 \quad x_3^* = 7 \quad x_4^* = 0 \quad x_5^* = 8 \quad x_6^* = 3$

$x_7^* = 11$

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2) Goutte Location problem

- French company manufacturing and distributing soft drinks

- Unexpected increase of its sales: new plants (design of logistics system from scratch)

- Supply costs: negligible (water and sugar extracts)

- Transportation costs: product independent, and calculated as

$$\boxed{\alpha \cdot h_i \cdot d_{ij}} \qquad \forall\, i,j$$

α: 0,67

h_i: annual demand of i

d_{ij}: distance (in Km) between i and j (given)

Obs: it is the annual transportation cost for satisfying the entire demand

Input data

- f_j : fixed cost for opening at j,
  $\forall j \in \underbrace{\{\text{Tolosa, Nizza, Marsiglia, Lione, Limoges, Digione, Parigi, Brest}\}}_{J}$

  (see the Excel file)

- transportation costs $\forall i \in I$, $j \in J$,
  where $I = \{\text{Tolosa, Nizza} \ldots \}$

  (see the Excel file)

- h_i : demand of i, $\forall i \in I$

  (see the Excel file)

- C_j : capacity of a plant in j, $\forall j \in J$

  (see the Excel file)

The problem : decide where to locate the plants, and how to allocate the demand nodes, in such a way as to satisfy the plant capacities at a minimum cost

Additional requirement (Quality of Service requirement) (QoS): maximum distance demand node – plant ~~set~~ to 700 Km

Decision variables:

$$x_j = \begin{cases} 1 & \text{if we locate a plant at } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J$$

$$0 \le y_{ij} \le 1 \quad \forall i \in I, j \in J$$ allocation variables

I.L.P. model (variant of (FCLP))

$$\text{Min} \sum_{j \in J} f_j \cdot x_j + \sum_{i \in I} \sum_{j \in J} \underbrace{(\alpha \cdot h_i \cdot d_{ij})}_{\text{transportation costs}} \cdot y_{ij}$$

Subject to:

$$\sum_{j \in J} y_{ij} = 1 \qquad \forall i \in I$$

$$\sum_{j \in J} h_i \cdot y_{ij} \le C_j x_j \qquad \forall j \in J$$

$$x_j \in \{0, 1\} \qquad \forall j \in J$$

$$0 \le y_{ij} \le 1 \qquad \forall i \in I, j \in J$$

$$y_{ij} = 0 \qquad \boxed{\forall i \in I, j \in J \text{ s.t. } d_{ij} > 700}$$

QoS constraints

<u>Obs</u> : $\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I$

$0 \leq y_{ij} \leq 1 \quad \forall i \in I, j \in J$

allow each demand node to be served by more plants

<u>The optimal solution</u> :

- 3 located plants (in Tolose, Limoges and Brest)
- <u>single-source allocation</u>
- optimum (≡ minimum) total cost : 32.210, 93

< <u>Excel file available</u> >

3) Cornovaglia Set Covering Location problem

- Consortium of 10 villages in Cornovaglia ($I = J$)
- Location of fire stations
- Covering distance (D_c) : 15 Km
- F : fixed cost for locating a fire station ($F = 123{,}000$)
- c_{ij} : cost for moving from $i \in I$ to $j \in J$ (see Excel file)

The problem : decide where to locate fire stations, and assign villages to fire stations so as to "cover" each village (within 15 Km), by minimizing the total cost (fixed costs + transportation costs)

Decision variables :

$$x_j = \begin{cases} 1 & \text{if we locate at } j \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in J$$

$$y_{ij} = \begin{cases} 1 & \text{if we assign } i \text{ to } j \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in I, j \in J$$

I.L.P. model (mix of (SCLP) and (FCLP)):

$$\text{Min} \sum_{j \in J} F \cdot x_j + \sum_{i \in I} \sum_{j \in J} c_{ij} \cdot y_{ij}$$

Subject to:

$$\sum_{j \in N_i} y_{ij} \geq 1 \quad \forall i \in I$$

within $D_c = 15$ Km (given)

$$\sum_{i \in I} y_{ij} \leq 10 \cdot x_j \quad \forall j \in J$$

* linking constraints *

$$x_j \in \{0, 1\} \quad \forall j \in J$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J$$

The optimal solution :

- open 2 fire stations, in Trevilley and in Zemmor (this is the minimum number of facilities to cover all villages ; why ?)
- assign 5 villages to each station

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