Logistics

Lecture notes*

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Chapter 8

Basic heuristics to CVRP

There are two main classes of heuristics that can be used to solve the Capacitated Vehicle Routing Problem (CVRP): classical heuristics (the mostly used ones), and metaheuristics, that require a deep analysis of the "most promising" solutions within the solution space (this kind of heuristics achieve better quality solutions but require a larger computational effort).

Classical heuristics include:

1. *constructive heuristics*: they construct a feasible solution step by step, without any improving phases;

2. *two-phase heuristics*: usually, in phase 1 the set of the nodes is partitioned into subsets (i.e. clusters), one for each tour; in phase 2, a tour is determined for each cluster. These two-phase heuristics are also called "cluster-first, route-second". Other heuristics, not considered in these notes, classifiy as "route-first, cluster-second";

3. *improvement heuristics*: given a current solution (e.g. obtained by the previous two kinds of heuristics), they perform steps to improve it (*local search*, *taboo search*. . .).

8.1 The Savings algorithm (by Clarke & Wright)

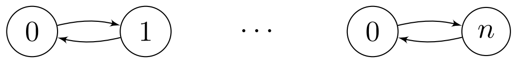
The method presented in this section is a constructive heuristic. The basic idea is the following: given two feasible routes, the cost variation if we merge them into a unique (feasible) route, as depicted in Figure 8.1, is:

$$s_{ij} = c_{i0} + c_{0j} - c_{ij}.$$

s_{ij} is called a *saving* since, if it is greater than zero, then it indicates an overall cost reduction.

The algorithm starts with n routes and, by using the $\{s_{ij}\}$, at each step it merges two routes, until it has obtained K feasible routes (recall that K denotes the cardinality of the fleet of the vehicles). In detail, the algorithm proceeds as follows:

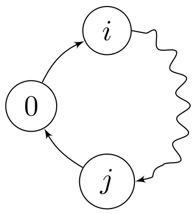
1. **Initial step**: determine n routes, one per customer:



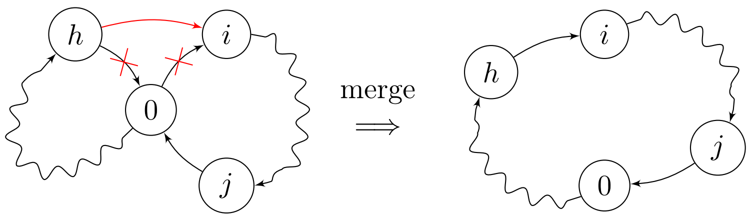
   and compute $s_{ij} = c_{i0} + c_{0j} - c_{ij}, \ \forall i, j = 1, \dots n, i \neq j$;

2. **Merge step**:

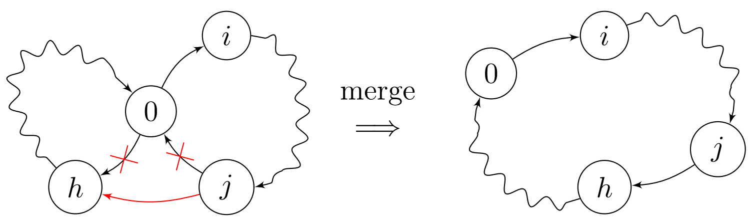
   - select one current route:



   - determine the largest s_{hi} such that by merging the current route with the one containing node h, as indicated below, we get a feasible route:



   - or determine the largest s_{jh} such that by merging the current route with the one containing node h, as indicated below, we get a feasible route:



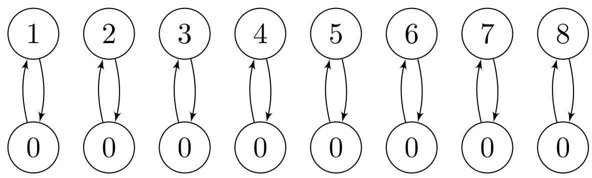
   - compute again the $\{s_{ij}\}$ if a merge operation has been performed.

3. Repeat Step 2 until K feasible routes will be obtained.

An example

Let $n = 8, K = 2$, the capacity of the vehicles $C = 8$, and the demands of the customers be given by $d = (1, 3, 2, 1, 2, 1, 1, 2)$.

1. **Initial step**: create 8 feasible routes like the following ones:



   and compute $\{s_{ij}\}$, $\forall i, j = 1, \dots 8, i \neq j$.

2. **Merge step**: select one current route, e.g. ⓪→②→⓪, determine the largest s_{h2} or the largest s_{2h} (with node h belonging to a different route), and merge the routes including h and 2 (if a feasible route will be obtained).

   For instance, assume that $s_{h2} = s_{52} = 1+2-1 = 2$ is the largest saving involving node 2. Since a feasible tour is obtained by merging the considered route with ⓪→⑤→⓪ (in fact, the total demand in the new tour is 5, and the capacity of the vehicles is 8), then perform the merge operation as depicted in Figure 8.2. We now have a solution composed of $n-1 = 7$ feasible routes.

3. Repeat Step 2 until $K = 2$ feasible routes will be obtained.

8.2 Sequential insertion heuristic by Mole & Jameson

The sequential insertion heuristic is a constructive heuristic, too. The idea underlying it is to start with a feasible tour ⓪→ⓤ→⓪ for some customer u, and at each step to try to extend the current tour by including an additional customer h, as depicted in Figure 8.3. In detail, the algorithm performs the following steps:

1. Initial step: compose a feasible route of the form ⓪→ⓤ→⓪ for some customer u not belonging to any route.

2. Compute $\alpha(r, h, s) = c_{rh} + c_{hs} - c_{rs}$ for each (r, s) in the route, and $\alpha^*(h) = \min_{(r,s)} \alpha(r, h, s)$.

3. Insertion step: insert node h with minimum $\alpha^*(h)$, if a feasible route will be obtained.

4. If no node can be inserted into the selected route, then go to Step 1 (if some isolated nodes exist), otherwise go to Step 2.

5. If K feasible routes have been obtained, then stop.

Notice that the K found feasible routes must include all the nodes, i.e. they must form a solution to CVRP.

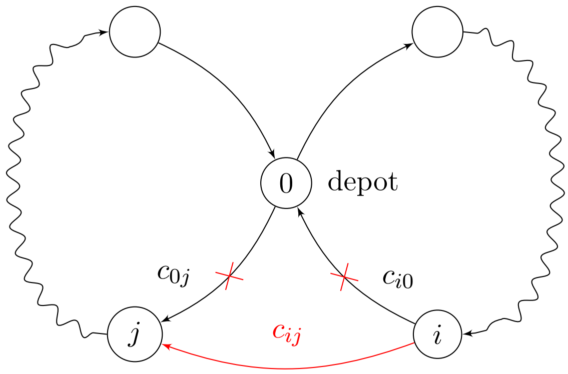


Figure 8.1: Merging two feasible routes in the Savings algorithm

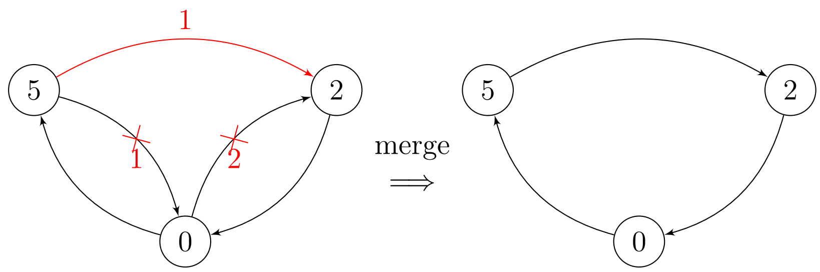


Figure 8.2: Merging route ⓪→②→⓪ with ⓪→⑤→⓪

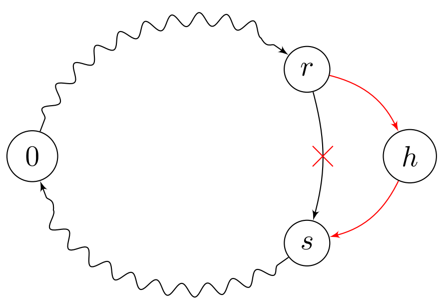


Figure 8.3: Including h into the current route

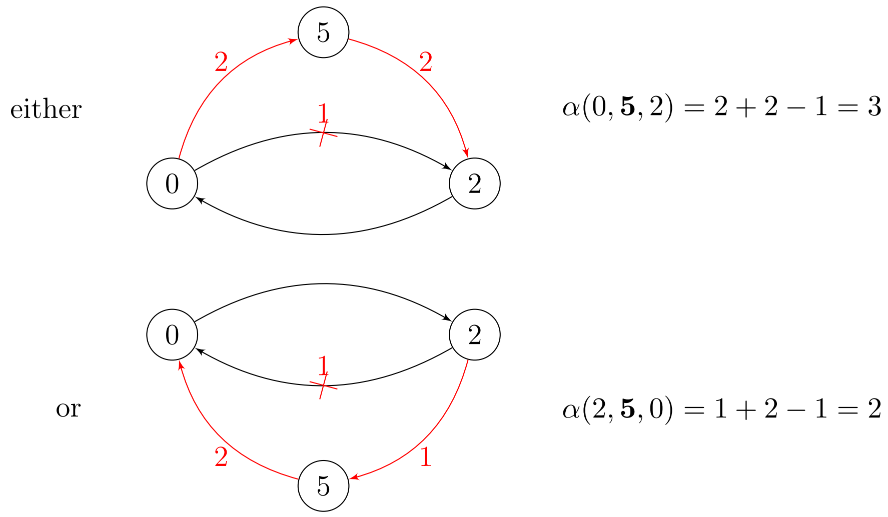


Figure 8.4: Insertion of customer 5 into the route ⓪→②→⓪

An example

Consider the same instance as before, i.e. $n = 8, K = 2, C = 8$ and $d = (1, 3, 2, 1, 2, 1, 1, 2)$.

1. Initial step: select a feasible route composed of a single customer not belonging to any tour, e.g. ⓪→②→⓪.
2. For each customer h not belonging to any route, find the minimum insertion cost. If, for instance, $h = 5$, and the costs are the ones shown in Figure 8.4, then the minimum insertion cost is $\alpha^*(5) = 2$.
3. Insertion step: insert the node h, which minimizes $\alpha^*(h)$, if this produces a feasible route. In the case of the example, ⓪→②→⑤→⓪ is indeed a feasible route.
4. If no node can be added to the selected route, then go to Step 1 (i.e. select another route), otherwise go to Step 2, by iterating until $K = 2$ feasible routes will be obtained.

8.3 Bramel & Simchi-Levi's heuristic

This is a two-phase heuristic, and the two phases are as follows:

1. *cluster determination*: in order to find the K subsets of nodes served by the K vehicles, the heuristic solves a capacitated p-median location problem, with $p = K$ and $I = J$. This will be better described next.

2. *tour determination*: for each cluster found in the previous phase, the heuristic uses a constructive heuristic for the VRP to determine the route of the vehicle within the cluster.

An example

Let $n = 6, K = 2, C = 5$ and $d = (1, 2, 2, 3, 1, 1)$. Also, let

$$[c_{ij}] = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & 0 & & & & & \\ \mathbf{2} & 1 & 0 & & & & \\ \mathbf{3} & 1 & & 0 & & & \\ \mathbf{4} & & & & 0 & & \\ \mathbf{5} & & & & 1 & 0 & \\ \mathbf{6} & & & & 1 & & 0 \end{array}$$

whereas $c_{ij} = 2$ for all the other arcs linking pairs of demand nodes, and for all the arcs incident to the depot, i.e. node 0.

In phase 1, the heuristic solves the instance of the capacitated 2-median problem presented in Model 8.1. The optimal solution to this problem, depicted in Figure 8.5, can be found by inspection and, by just considering the components other than zero, it is:

$$x_1^* = x_4^* = 1, \quad y_{11}^* = y_{21}^* = y_{31}^* = y_{44}^* = y_{54}^* = y_{64}^* = 1$$

$$\begin{array}{ll} \min \sum_{i \in \{1,\dots 6\}} \sum_{j \in \{1,\dots 6\}} c_{ij} y_{ij} & \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 2 & \text{location} \\ \left.\begin{array}{l} y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} = 1 \\ y_{21} + y_{22} + y_{23} + y_{24} + y_{25} + y_{26} = 1 \\ \vdots \\ y_{61} + y_{62} + y_{63} + y_{64} + y_{65} + y_{66} = 1 \end{array}\right\} & \text{assignment} \\ \left.\begin{array}{l} y_{11} + 2y_{21} + 2y_{31} + 3y_{41} + y_{51} + y_{61} \leq 5x_1 \\ \vdots \\ y_{16} + 2y_{26} + 2y_{36} + 3y_{46} + y_{56} + y_{66} \leq 5x_6 \end{array}\right\} & \text{linking} \\ x_1, x_2, \dots y_{11}, \dots \in \{0, 1\} & \end{array}$$

Model 8.1

Therefore, according to the heuristic, customers $\{1, 2, 3\}$ will be served by a vehicle, and customers $\{4, 5, 6\}$ by another vehicle.

In phase 2 (determination of a tour for each cluster), we can use a constructive heuristic for the VRP, separately for each cluster. For example, let us use the sequential insertion heuristic presented in Section 8.2. Figure 8.6 shows the steps performed by the sequential insertion heuristic to obtain the feasible routes for the two clusters computed in phase 1. Note that the feasibility with respect to the capacity is guaranteed by phase 1.

So, the solution returned by the Bramel & Simchi-Levi's heuristic is the one in Figure 8.7, with cost 14 (note that there is no guarantee of optimality).

References P. Toth and D. Vigo (2002): Chapter 5

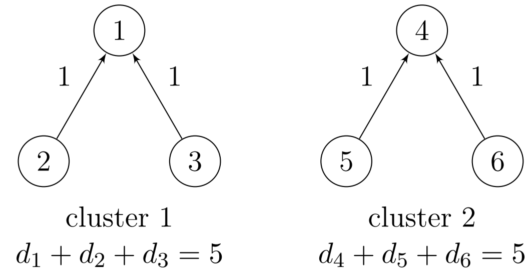


Figure 8.5: Cluster determination for the example

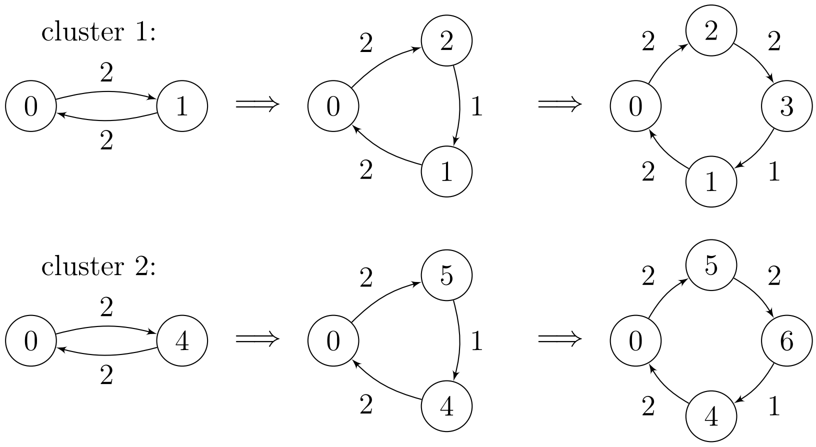


Figure 8.6: Route determination for the example

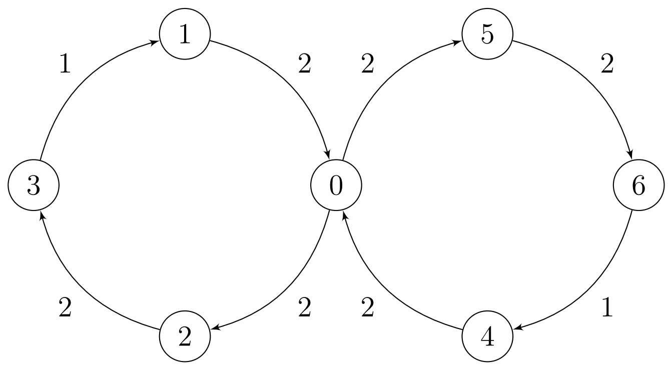


Figure 8.7: Heuristic solution to the example

Textbooks

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