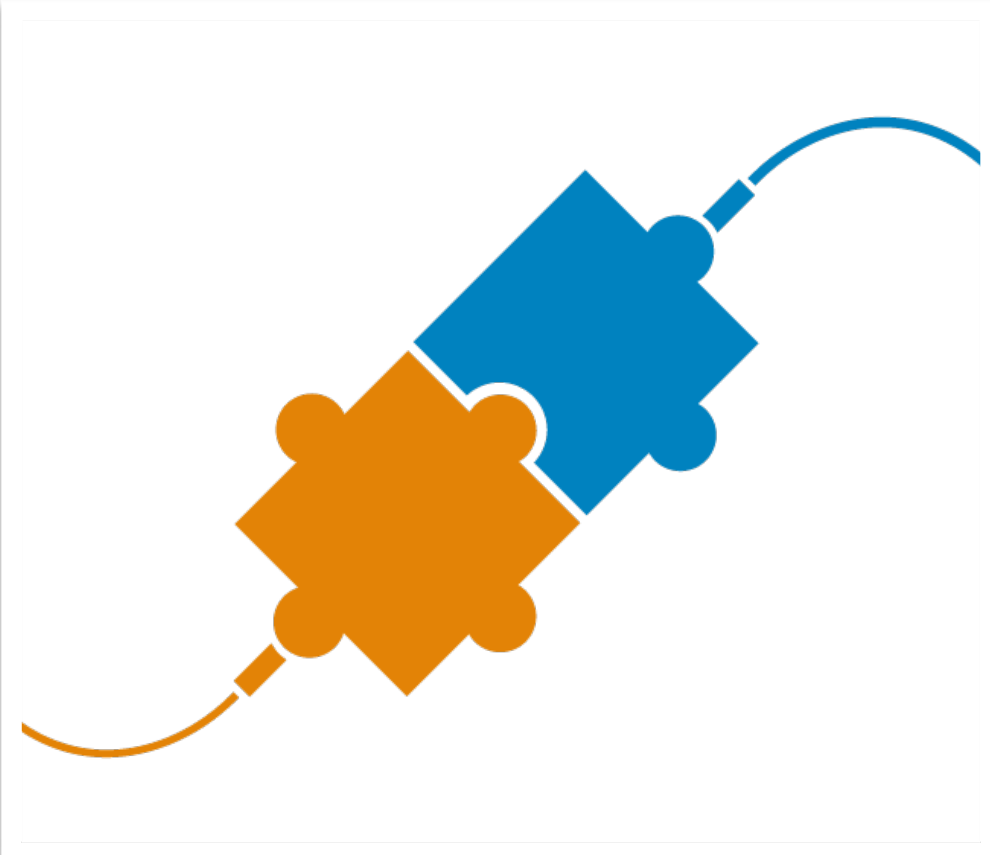


http://didawiki.di.unipi.it/doku.php/magistraleinformatica/psc/start

PSC 2023/24 (375AA, 9CFU)

Principles for Software Composition

Roberto Bruni
http://www.di.unipi.it/~bruni/

01 - Introduction

English vs Italian

Classes

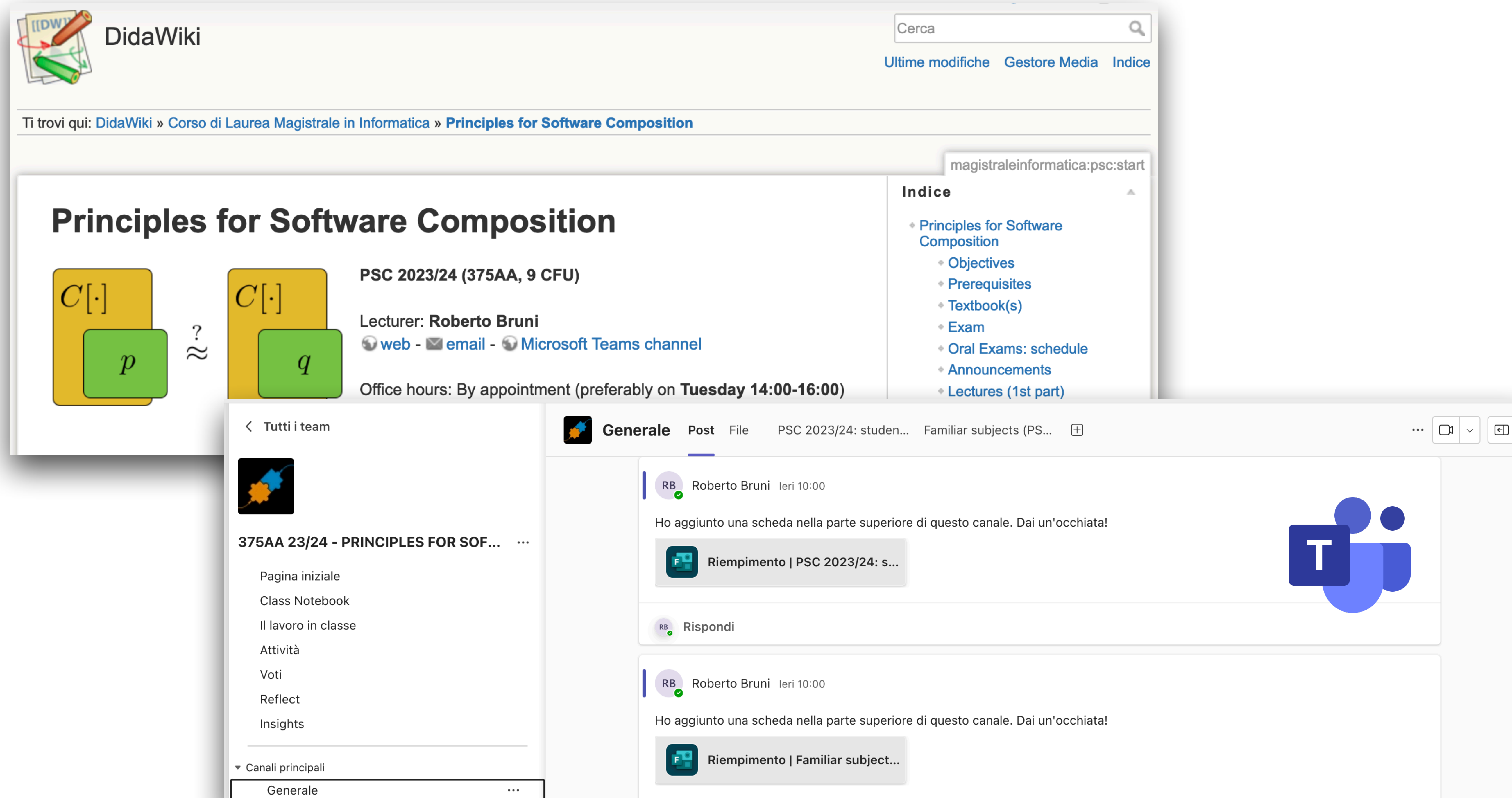
In presence, every

Tuesday: 16:00-18:00, C1

Thursday: 16:00-18:00, M1

Friday: 14:00-16:00, C1

Course material



Who am I?

http://www.di.unipi.it/~bruni

bruni@di.unipi.it

Office hours: by appointment
preferably
Tuesday 14:00-16:00

Research topics (theses?)

False alarm detection in Abstract Interpretation

Formal approaches to code obfuscation

Quantum Computation and concurrency models

Modelling and analysis of biological systems

Graphical specification languages

Algebraic approaches to structured graphs

Rewrite rules for reversible languages

Who are you?

Forms

First Name:

Last Name:

Enrollment number:

email:

Bachelor degree:

MSc course of enrollment:

Please fill the form!

Who are you?

Forms

First Name: John
Last Name: Smith
Enrollment number: 123456
email: john.smith@email.com
Bachelor degree: Comp. Sci., Pisa, IT
MSc course of enrollment: Comp. Sci. - SW

Please fill the form!

Interaction protocol

Do you know what are the 3 possible answers to the question "does my program c satisfy the property ψ?"

- ○ yes
- ○ no
- ○ don't know

(you MUST select one of them to answer questions in these classes)

The Course

Some quotes

Computer science is no more about computers than astronomy is about telescopes
- Edsger W. Dijkstra

Studying programming languages without formal semantics would be like studying physics without math
- from the web

All models are wrong, but some are useful
- George Box

Subjects are divided in two categories:
1) *too difficult matters, that CANNOT be studied*
2) *easy matters, that DO NOT NEED to be studied*

- back of a t-shirt

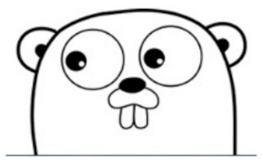
Objectives

Programming paradigms
(imperative, declarative, higher order, concurrent, mobile, stochastic)

SWI Prolog

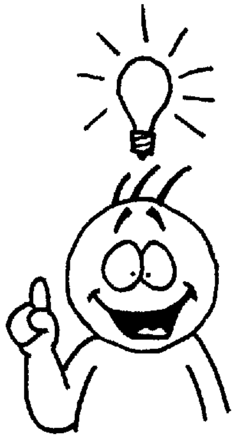
Google Go!

ERLANG



Mathematical frameworks
(concrete & abstract)
(domains, inference rules, transition systems, λ-calculus, process algebras)

Understand
(recursion, semantics, compositionality)



Reason
(induction, modal and temporal logics, behavioural and logical equivalences)

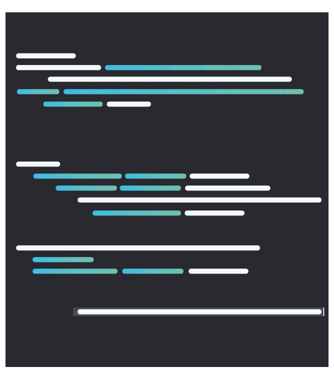
Explain
(correctness, compliance, performance)

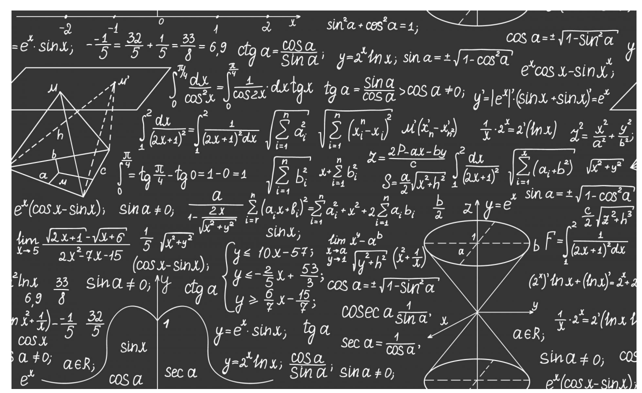
The approach

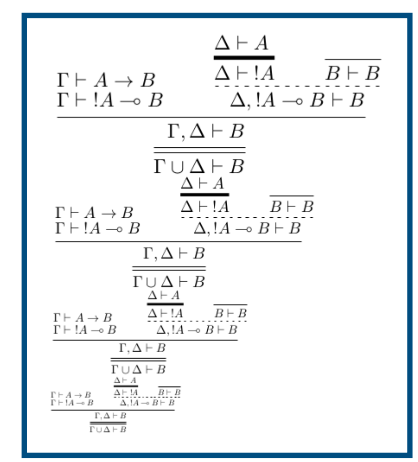
(in their simplest form,
still Turing equivalent)

programming paradigms — mathematical frameworks — meta-properties + proof techniques

(for all programs
or just
some classes of programs)

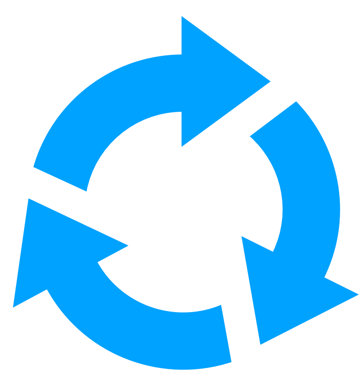






models

programs



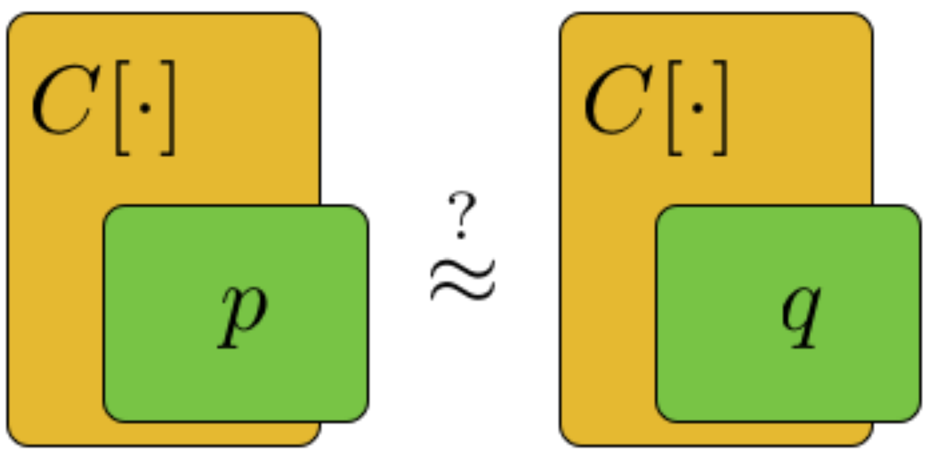
specifications

Key question

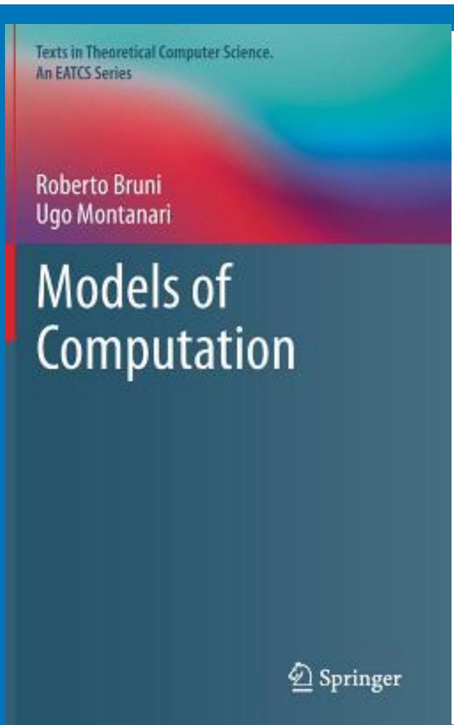
Given two programs *p* and *q*:

Do they behave the same?

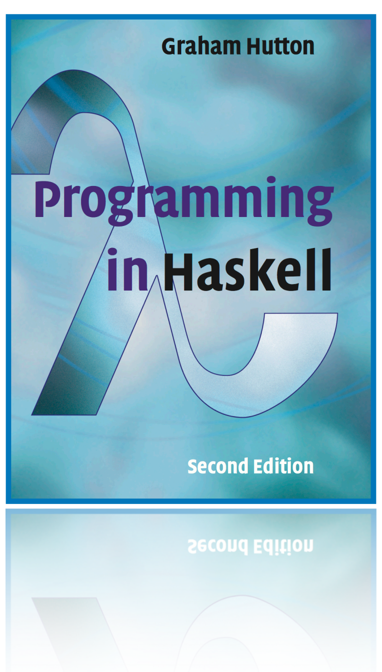
Is it safe to replace one with the other in any context?

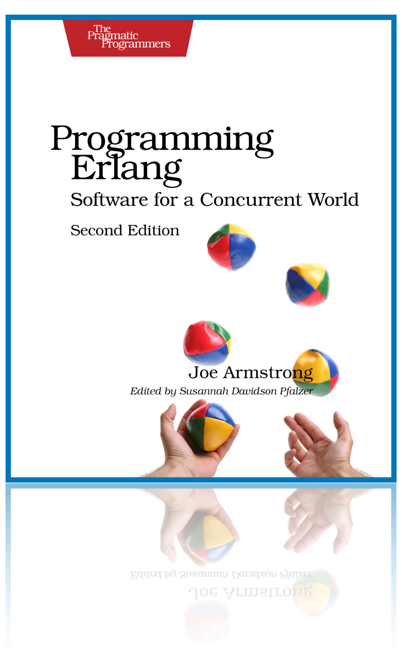


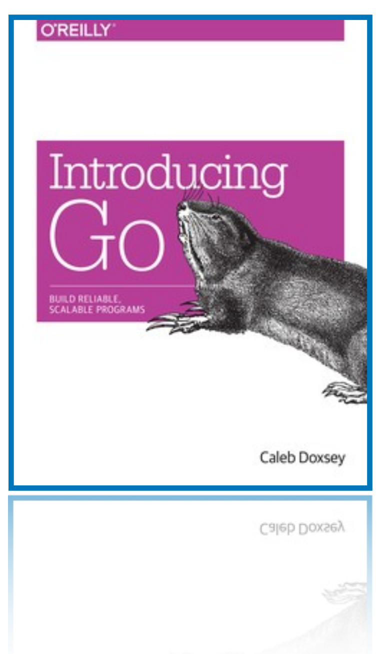
Textbooks

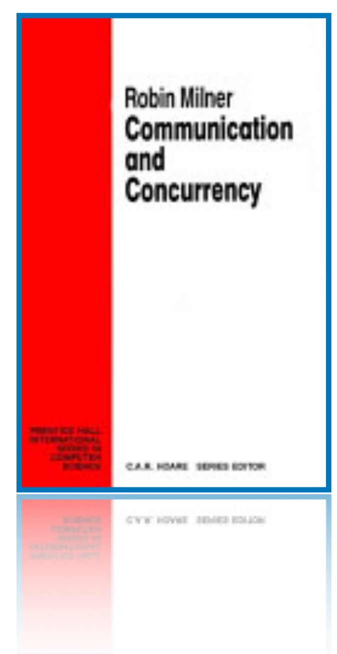


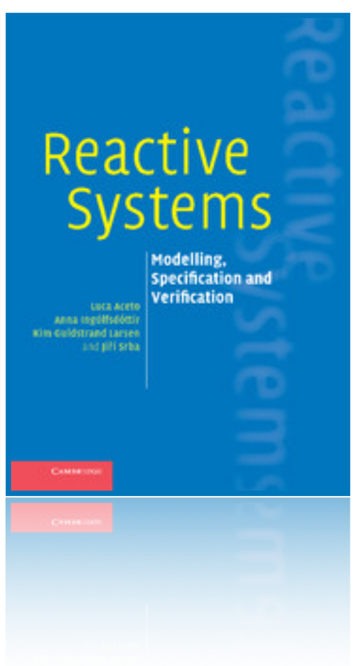
Roberto Bruni and Ugo Montanari
Models of Computation
Texts in Theoretical Computer Science (an EATCS series)
https://www.springer.com/book/9783319428987

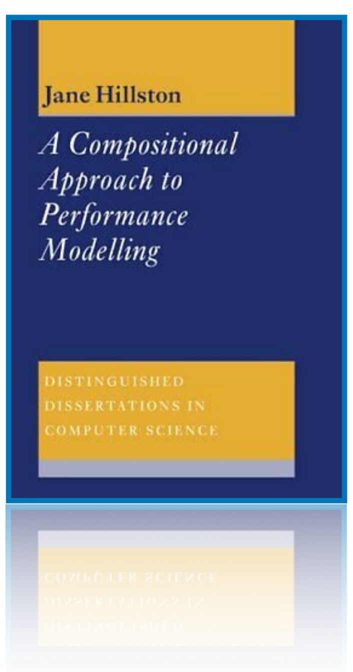








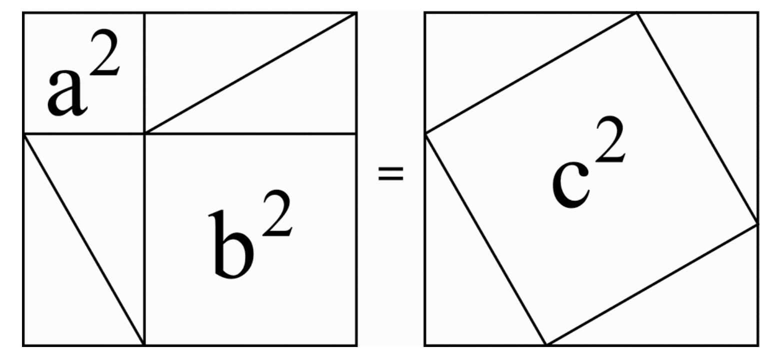




Course activities

attend classrooms:
ask questions!
(sleep quietly)

learn theorems:
(drink many coffees)



do some thinking:
solve ALL your
homeworks
(at least try to)

give the exam:
time for a party!

Be proactive!

Let's spell out definitions together

```
% find the least (non-unitary) divisor p of n>1
p := 0;
x := 2;
while (.............................................) do {
    if ( n%x == 0 ) then {
        p := x;
    } else {
        x := x + 1;
    }
}
```

Be proactive!

Correct me if I'm wrong

```
% find the index of the last occurrence of n in a
i := length(a)-1;
while ( i>0 && n!=a[i] ) do {
   i := i-1;
}
```

Be proactive!

Sometimes tricky questions!

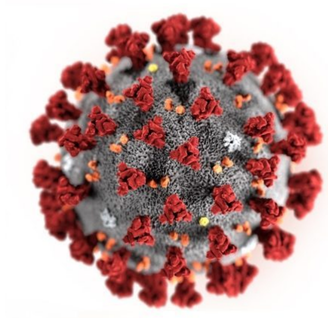
Can you find the the mistake?

1 2 3 4 5 6 7 8 9

Exam

In past years, the evaluation was based on written and oral exams.

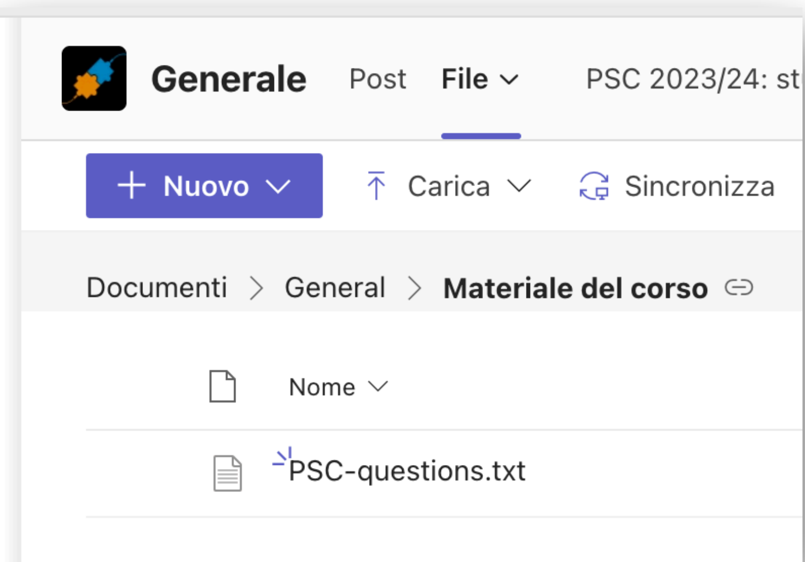
Since the covid-19 emergency, and for the current period, the evaluation will be solely based on a final oral exam.



Registration to exams is mandatory:
https://esami.unipi.it/esami

The exam will typically consist of:
1. three to four preliminary questions
2. one exercise (analogous to past written exams)
3. redoing one of the proofs seen in the course
4. some additional questions

The list of preliminary questions is available on Microsoft Teams, in the File tab (`PSC-questions.txt`)



A sample exam

What is a complete partial order?

What are the rules of the type system of HOFL?

How is iteration achieved in CCS?

Why only positive normal forms are considered in the mu-calculus?

Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \mathbf{if}\ x\ \mathbf{then}\ (x, \mathbf{fst}(f\ x))\ \mathbf{else}\ (\mathbf{snd}(f\ x), x)$$

1. Find the principal type of t.
2. Find the denotational semantics of t.

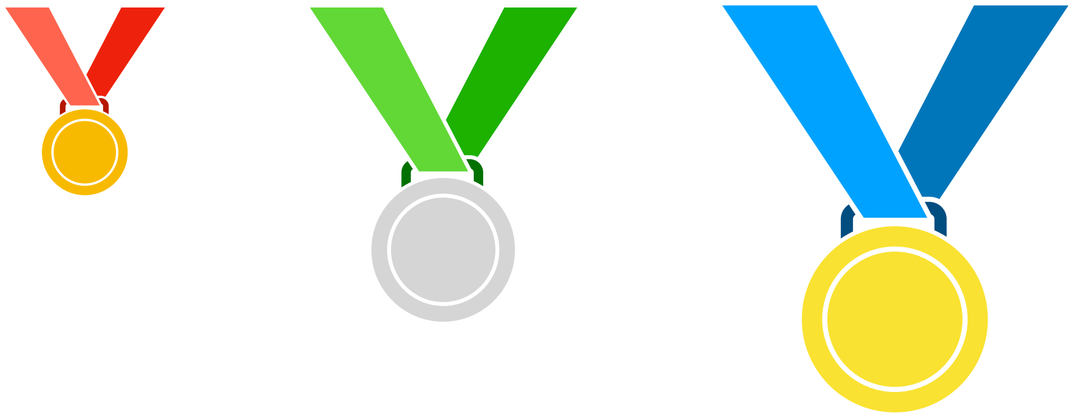
Prove the Switch Lemma

Given the initial state distribution and a DTMC,
how do we compute the state distribution at time 3?

Badges

No mid-terms
No self-evaluation tests
During the course: some “badge” exercises

Submit your solutions by email to earn
bronze / silver / gold badges
(no extra scores, but be proud of yourselves)



Prerequisites

Basic set theory

$\varnothing$ $A \cap B$ $A \cup B$ $A \setminus B$ $\overline{A}$

$a \in A$ $A \subset B$ $A \subseteq B$ $A \times B$

$a \notin A$ $A \not\subseteq B$ $A \cap B = \emptyset$

$\mathbb{N}$ $\mathbb{Z}$ $\mathbb{Q}$ $\mathbb{R}$ $\mathbb{B}$

$N \subseteq \mathbb{N}$ $N \in \wp(\mathbb{N})$ $S \subseteq \wp(\mathbb{N})$

Prerequisites

Basic set theory: functions, relations

$$f : A \to B$$

$$R \subseteq A \times B$$

functions as relations

$$R_f \triangleq \{(a, f(a)) \mid a \in A\}$$

sets as functions
(characteristic function)

$$f_N : \mathbb{N} \to \mathbb{B}$$

$$f_N(n) \triangleq \begin{cases} 1 & n \in N \\ 0 & \text{otherwise} \end{cases}$$

$$N = \{n \mid f_N(n) = 1\}$$

Prerequisites

First order logic

ff false 0 F

tt true 1 T

$P \wedge Q$ $P \vee Q$ $\neg P$

$\exists x.\ P(x)$ $\forall x.\ P(x)$ $P \Rightarrow Q$ $P \Leftrightarrow Q$

meaning of implication!

$$P \Rightarrow Q$$

$$Q \vee \neg P$$

$$\neg Q \Rightarrow \neg P$$

order of quantifiers matters!

$$\forall n \in \mathbb{N}.\ \exists m \in \mathbb{N}.\ n < m$$

$$\exists m \in \mathbb{N}.\ \forall n \in \mathbb{N}.\ n < m$$

Prerequisites

Strings and context-free grammars

Alphabet A $\quad A^n \triangleq \underbrace{A \times \cdots \times A}_{n} \quad A^* \triangleq \bigcup_{n \in \mathbb{N}} A^n$

$$
\begin{aligned}
\mathbb{B} &= \{0, 1\} \\
\mathbb{B}^0 &= \{\epsilon\} \\
\mathbb{B}^1 &= \{0, 1\} \\
\mathbb{B}^2 &= \{00, 01, 10, 11\} \\
\mathbb{B}^3 &= \{000, 001, 010, 011, 100, 101, 110, 111\} \\
&\ldots \\
\mathbb{B}^* &= \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}
\end{aligned}
$$

Prerequisites

Strings and context-free grammars

Alphabet A $\qquad A^n \triangleq \underbrace{A \times \cdots \times A}_{n} \qquad A^* \triangleq \bigcup_{n \in \mathbb{N}} A^n$

$$\mathbb{B}^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$$

$$\mathsf{A} ::= \epsilon \mid 0\ \mathsf{A} \mid 1\ \mathsf{B}$$

$$\mathsf{B} ::= 0\ \mathsf{B} \mid 1\ \mathsf{A}$$

$$\mathsf{A} \to 0\ \mathsf{A} \to 0\ 1\ \mathsf{B} \to 0\ 1\ 1\ \mathsf{A} \to 0\ 1\ 1\ \epsilon = 0\ 1\ 1$$

$\mathcal{L}(\mathsf{A}) = ?$ $\qquad \mathcal{L}(\mathsf{B}) = ?$

Prerequisites

Inductive and recursive definitions

$$
\begin{aligned}
0! &\triangleq 1 \\
(n+1)! &\triangleq n! \cdot (n+1)
\end{aligned}
$$

$$
\begin{aligned}
A^0 &\triangleq \{\epsilon\} \\
A^{(n+1)} &\triangleq A \times A^n
\end{aligned}
$$

$$
f(n) \triangleq \begin{cases} 1 & \text{if } n \leq 1 \\ f(n/2) & \text{if } n > 1 \wedge n\%2 = 0 \\ f(3n+1) & \text{otherwise} \end{cases}
$$

$$
f(12) = f(6) = f(3) = f(10) = f(5) = f(16) = f(8) = f(4) = f(2) = f(1) = 1
$$

Prerequisites

Conjectures vs theorems

a natural number ***p*** is **prime**
if it cannot be written as the product of two smaller numbers

n	Is n prime?	$2^n - 1$	Is $2^n - 1$ prime?
2	yes	3	yes
3	yes	7	yes
4	no: $4 = 2 \cdot 2$	15	no: $15 = 3 \cdot 5$
5	yes	31	yes
6	no: $6 = 2 \cdot 3$	63	no: $63 = 7 \cdot 9$
7	yes	127	yes
8	no: $8 = 2 \cdot 4$	255	no: $255 = 15 \cdot 17$
9	no: $9 = 3 \cdot 3$	511	no: $511 = 7 \cdot 73$
10	no: $10 = 2 \cdot 5$	1023	no: $1023 = 31 \cdot 33$

Prerequisites

Conjectures vs theorems

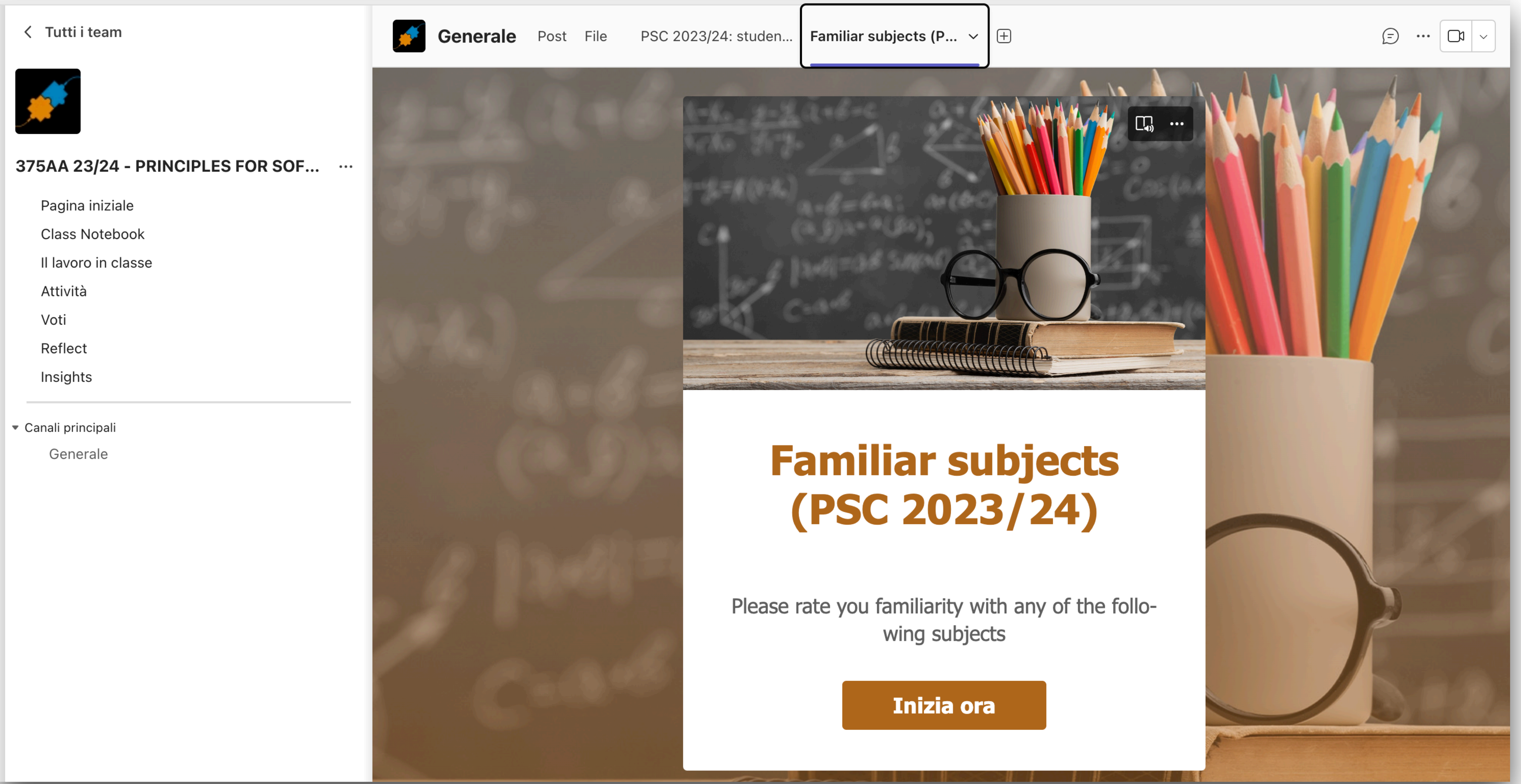
if p is prime
then $2^p - 1$ is prime

if $n > 1$ is not prime
then $2^n - 1$ is not prime

Use any mean to prove or disprove the above conjectures

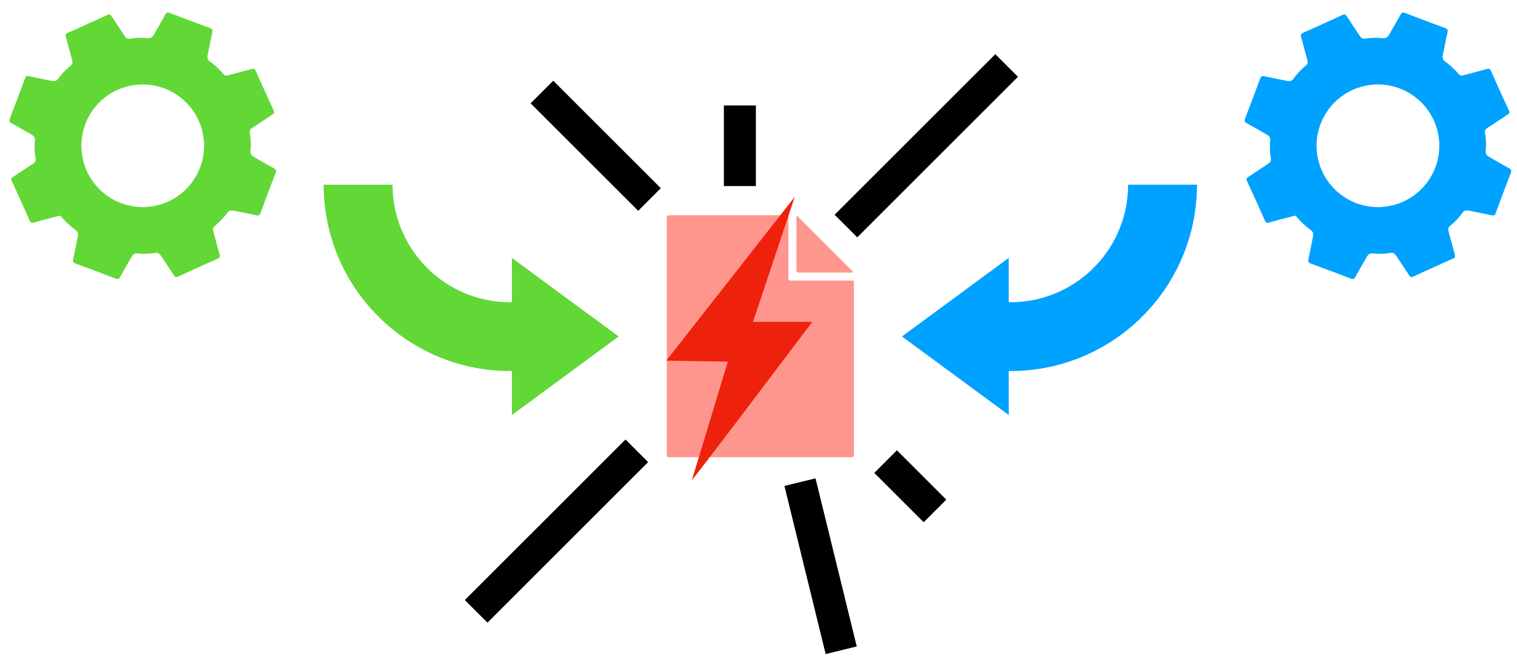
Your background?

Please fill the form about “Familiar subjects”



An Appetiser

The problem



Two concurrent processes share a single-use resource

They can communicate using shared memory

We want to guarantee that there are no conflicts
when the processes access the resource

No strict alternation of naive turn taking is imposed

Peterson's mutual exclusion algorithm (1981)

```
% Two processes P1, P2
% Two boolean variables b1, b2 (both initially false)
% when Pi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Pk has priority over the other process
%
% Process P1 in pseudocode
while (true) {
    ...                            % non critical section
    b1 := true ;                   % P1 wants to enter the critical section
    k := 2 ;                       % P1 gives priority to the other process
    while (b2 && k==2) skip ;      % P1 waits its turn
    ...                            % P1 enters the critical section
    b1 := false                    % P1 leaves the critical section
}

% Process P2 is analogous to P1
```

Which question?

Does Peterson's algorithm work?

What does it mean that "it works"? What do we expect?

(Progress)
If the resource is available, no process is forced to wait

(Bounded Waiting)
No process will wait forever for the resource
(otherwise the easiest solution is no one gets in)

(Mutual Exclusion)
P1 and P2 are never in the critical section at the same time

Hyman’s mutual exclusion algorithm (1966)

```
% Two processes H1, H2
% Two boolean variables b1, b2 (both initially false)
% when Hi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Hk has priority over the other process
%
% Process H1 in pseudocode
while (true) {
    ...                          % non critical section
    b1 := true ;                 % H1 wants to enter the critical section
    while (k==2) {               % while H2 has priority
        while (b2) skip ;        % H1 waits
        k := 1;                  % H1 sets priority to itself
    }
    ...                          % H1 enters the critical section
    b1 := false                  % H1 leaves the critical section
}

% Process H2 is analogous to H1
```

The question

Does Peterson's algorithm satisfy mutual exclusion?

Does Hyman's algorithm satisfy mutual exclusion?

For the answers be patient and wait early-May lectures