

Principles for software composition 2023/24

01 - Unification, goal-oriented derivations, mathematical induction

[Ex. 1] Let $\Sigma_0 = \{0\}$ and $\Sigma_1 = \{s\}$. Extend the logic program that defines the predicate $\text{sum} \in \Pi_3$ (seen in classroom) to define:

1. a predicate $\text{prod} \in \Pi_3$ for computing the product of two numbers;
2. a predicate $\text{pow} \in \Pi_3$ for computing the power;
3. a predicate $\text{div} \in \Pi_2$ for telling if the first argument divides the second.

[Ex. 2] Given the syntax in Ex. 1, solve the unification problems below

1. $G_1 \stackrel{\text{def}}{=} \{\text{prod}(s(x), y, s(z)) \stackrel{?}{=} \text{prod}(y, z, x)\}$
2. $G_2 \stackrel{\text{def}}{=} \{\text{pow}(x, s(y), x) \stackrel{?}{=} \text{pow}(s(y), z, z)\}$
3. $G_3 \stackrel{\text{def}}{=} \{\text{div}(x, s(y)) \stackrel{?}{=} \text{div}(z, x) , \text{div}(y, s(z)) \stackrel{?}{=} \text{div}(u, s(u))\}$

[Ex. 3] Given the logic programs in Ex. 1, write some possible goal-oriented derivations for the queries:

1. $\text{sum}(x, s(0), s(s(0)))$
2. $\text{prod}(s(s(0)), y, s(s(0)))$
3. $\text{div}(z, s(s(0)))$

[Ex. 4] Prove by mathematical induction that:

$$\forall n > 0. n^n \geq n!$$

[Ex. 5] Let

$$a_0 \stackrel{\text{def}}{=} 0 \quad a_{n+1} \stackrel{\text{def}}{=} 2a_n + n$$

Prove by mathematical induction that:

$$\forall n \in \mathbb{N}. a_n = 2^n - n - 1$$

[Ex. 6] Let F_i denote the i th Fibonacci number.

$$F_1 \stackrel{\text{def}}{=} 1 \quad F_2 \stackrel{\text{def}}{=} 1 \quad F_{n+2} \stackrel{\text{def}}{=} F_n + F_{n+1}$$

Prove by mathematical induction that:

$$\forall n > 0. \sum_{i=1}^n F_i = F_{n+2} - 1$$