Overview

Introduction

Modelling parallel systems

Linear Time Properties

- state-based and linear time view
- definition of linear time properties
- invariants and safety
- liveness and fairness ⟵

Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Liveness

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"whenever event ***b*** occurs then event ***a***
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e.g., every waiting process enters eventually
its critical section

which property type?

- Each philosopher thinks infinitely often.

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here: one just example for a formal definition of liveness

Definition of liveness properties

LF2.6-DEF-LIVENESS

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recall: $pref(E) =$ set of all finite, nonempty prefixes of words in E

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Examples:

- each process will eventually enter its critical section
- each process will enter its critical section infinitely often
- whenever a process has requested its critical section then it will eventually enter its critical section

Examples for liveness properties

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$$\forall i \in \{1, \ldots, n\}\ \forall j \geq 0.\ wait_i \in A_j \longrightarrow \exists k > j.\ crit_i \in A_k$$

Recall: safety properties, prefix closure

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iff $\forall \sigma \in \left(2^{AP}\right)^{\omega} \backslash E \;\; \exists A_0 A_1 \ldots A_n \in \mathit{pref}(\sigma)$ s.t.

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$$pref(\sigma) = \text{set of all finite, nonempty prefixes of } \sigma$$

$$pref(E) = \bigcup_{\sigma \in E} pref(\sigma)$$

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Show that:

- $E = SAFE \cap LIVE$
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Being safe and live

Which LT properties are both
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If E is a safety property too, then $\mathit{cl}(E) = E$.

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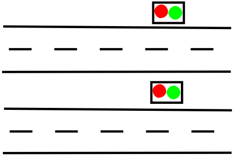
If E is a safety property too, then $cl(E) = E$.
Hence $E = cl(E) = (2^{AP})^{\omega}$.

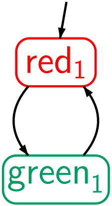
Observation

liveness properties are often violated
although we expect them to hold

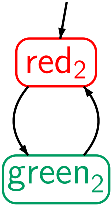
Two independent traffic lights



light 1



light 2



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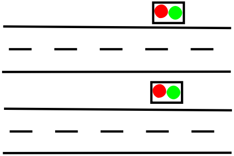

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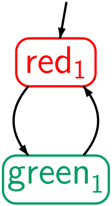
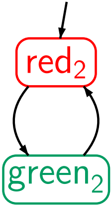


light 2



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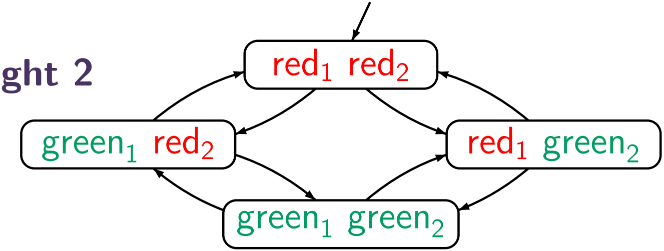

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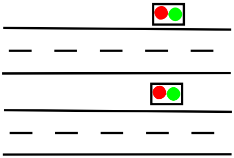

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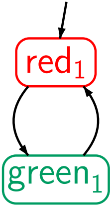

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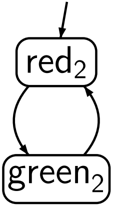

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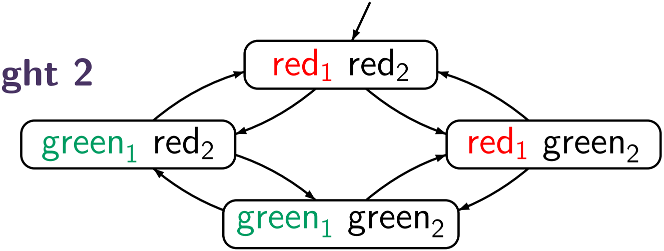
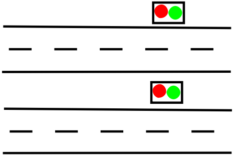


light 1 ||| light 2 $\not\models$ "infinitely often ***green$_1$***"

Two independent traffic lights



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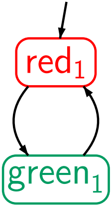

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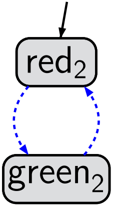

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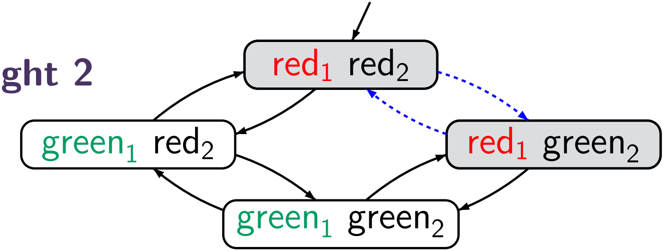

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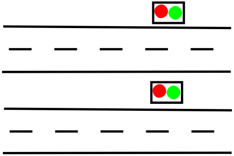

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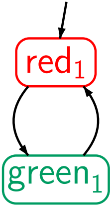

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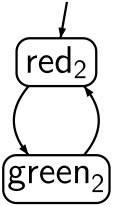

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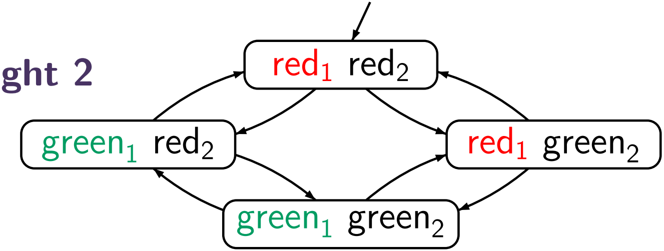

light 1 ||| light 2 $\not\models$ "infinitely often ***$green_1$***"

although **light 1** $\models$ "infinitely often ***$green_1$***"

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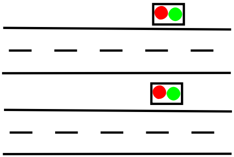

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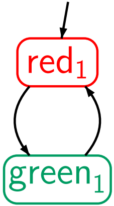

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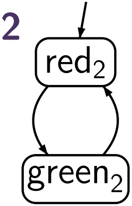

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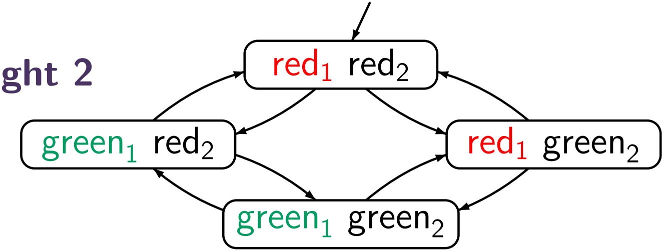

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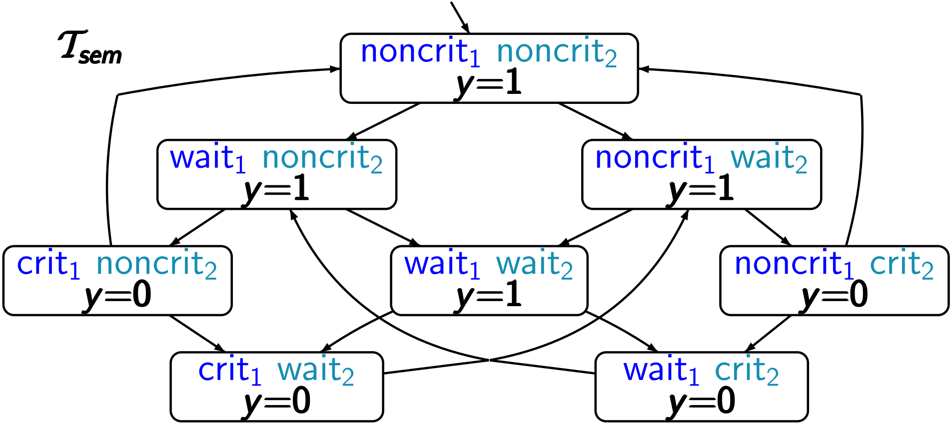
interleaving is completely time abstract **!**

Mutual exclusion (semaphore)



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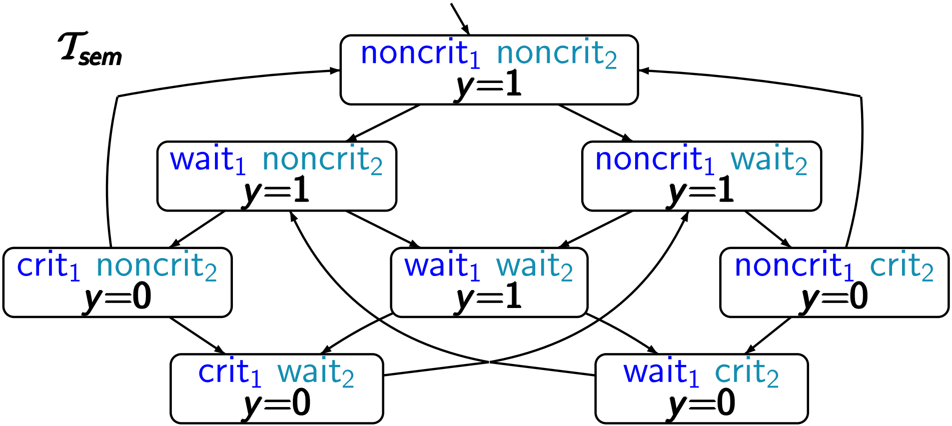

liveness property $\widehat{=}$ “each waiting process will eventually enter its critical section”

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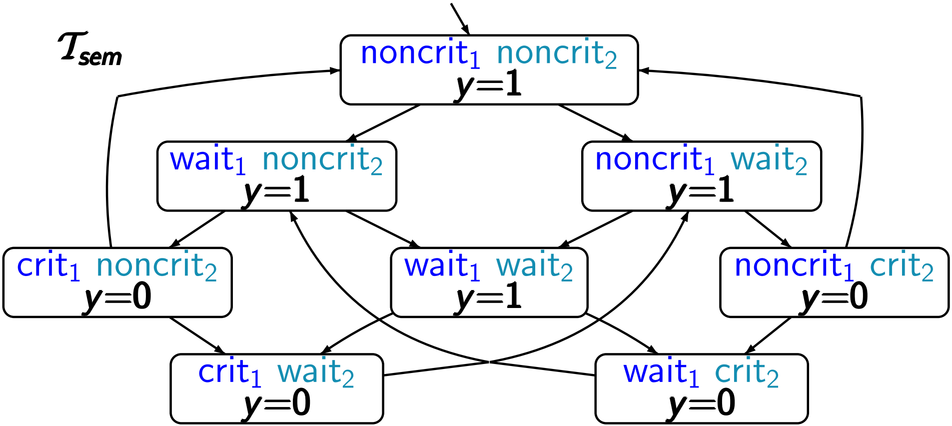

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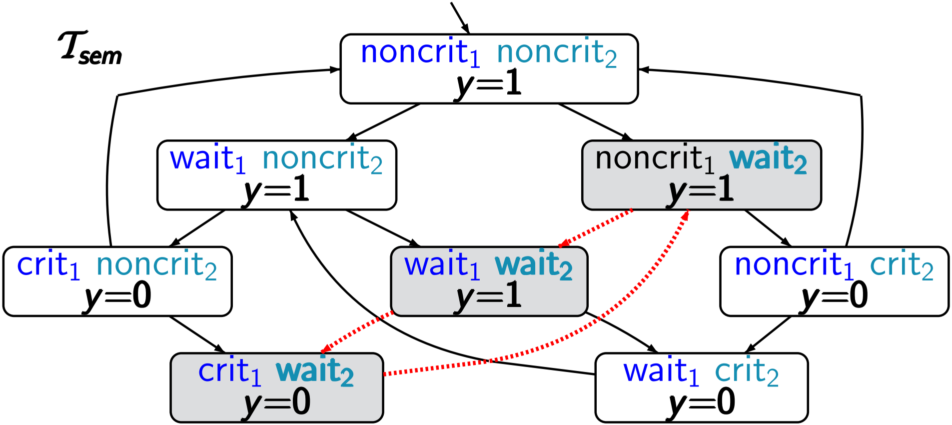

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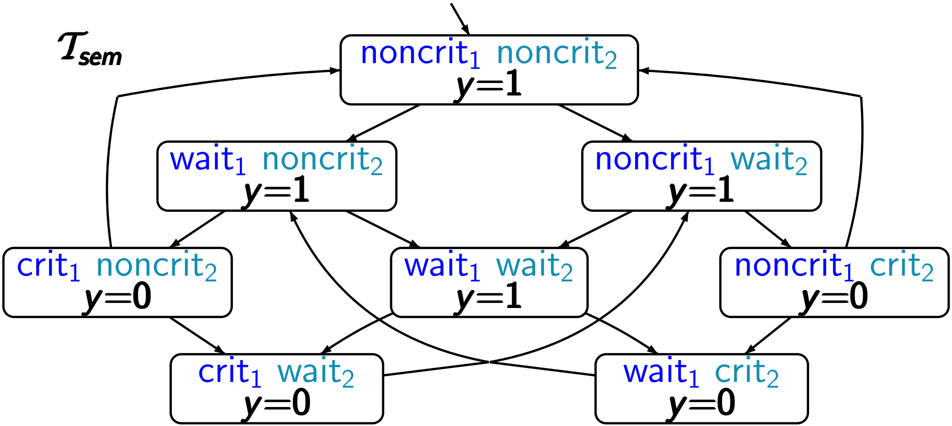

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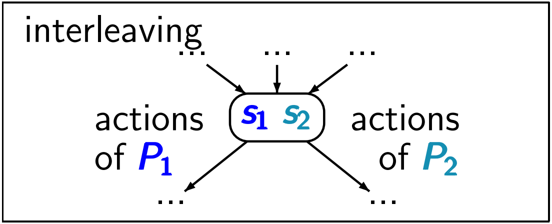
level of abstraction is too coarse !

Process fairness

LF2.6-5

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



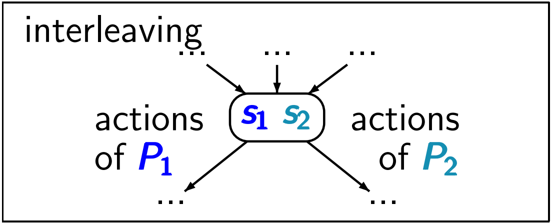
possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ...
P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ...

Process fairness

LF2.6-5

two independent
non-communicating
processes $P_1 \mid\mid\mid P_2$


interleaving

actions of P_1

s_1 s_2

actions of P_2

…

possible interleavings:

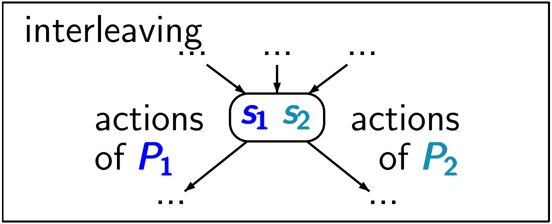
$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$

$P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ \ldots$

$P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ \ldots$

Process fairness

two independent non-communicating processes $P_1 ||| P_2$

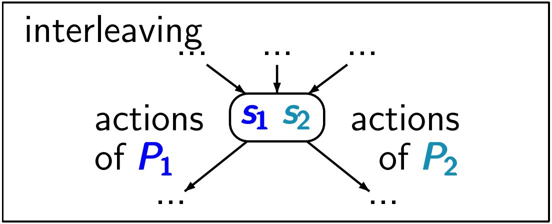


possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ... fair
P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ... fair
P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 ... unfair

Process fairness

two independent non-communicating processes $P_1 \;|||\; P_2$



possible interleavings:

$P_1 \; P_2 \; P_2 \; P_1 \; P_1 \; P_1 \; P_2 \; P_1 \; P_2 \; P_2 \; P_2 \; P_1 \; P_1 \; \ldots$ fair

$P_1 \; P_1 \; P_2 \; P_1 \; P_1 \; P_2 \; P_1 \; P_1 \; P_2 \; P_1 \; P_1 \; P_2 \; P_1 \; \ldots$ fair

$P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; P_1 \; \ldots$ unfair

process fairness assumes an appropriate resolution of the nondeterminism resulting from interleaving and competitions

Nuances of fairness

- unconditional fairness

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

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- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness, e.g.,

  every process that is continuously enabled
  from a certain time instance on,
  gets its turn infinitely often.

Fairness for action-set

LF2.6-7

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq \mathit{Act}$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

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we will provide conditions for

- unconditional A-fairness of ρ
- strong A-fairness of ρ
- weak A-fairness of ρ

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using the following notations:

$$Act(s_i) \;=\; \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

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$$\overset{\infty}{\exists} \;\widehat{=}\; \text{“there exists infinitely many ...”}$$

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and $\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

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using the following notations:

$$\begin{array}{rcl} Act(s_i) & = & \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\} \\ \overset{\infty}{\exists} & \widehat{=} & \text{"there exists infinitely many ..."} \\ \overset{\infty}{\forall} & \widehat{=} & \text{"for all, but finitely many ..."} \end{array}$$

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and $\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

- ρ is unconditionally A-fair, if

Fairness for action-set

LF2.6-7A

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

- ρ is unconditionally A-fair, if $\overset{\infty}{\exists}\, i \geq 0.\ \alpha_i \in A$

“actions in A will be taken infinitely many times”

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

- ρ is unconditionally A-fair, if $\overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$
- ρ is strongly A-fair, if

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is unconditionally A-fair, if $\overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$
- ρ is strongly A-fair, if

$$\overset{\infty}{\exists}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If infinitely many times some action in A is enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is weakly A-fair, if

$$\overset{\infty}{\forall} i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists} i \geq 0.\, \alpha_i \in A$$

"If from some moment, actions in A are enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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unconditionally A-fair $\Longrightarrow$ strongly A-fair

$\Longrightarrow$ weakly A-fair

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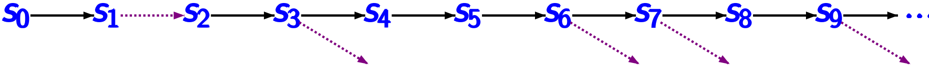
unconditionally A-fair $\Longrightarrow$ strongly A-fair

$\Longrightarrow$ weakly A-fair

Strong and weak action fairness

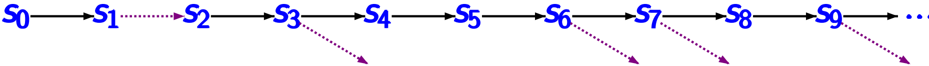
strong ***A***-fairness is *violated* if



- no ***A***-actions are executed from a certain moment
- ***A***-actions are enabled infinitely many times

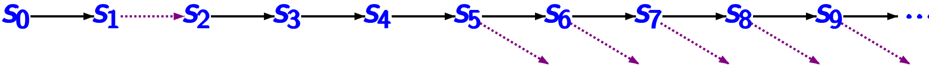
Strong and weak action fairness

strong ***A***-fairness is *violated* if



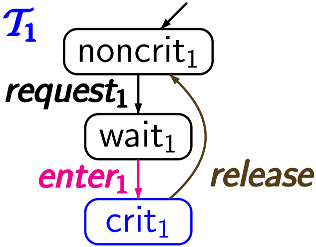
- no ***A***-actions are executed from a certain moment
- ***A***-actions are enabled infinitely many times

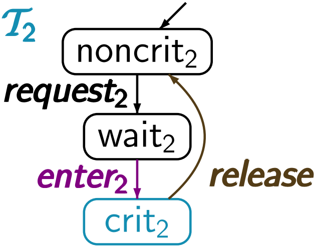
weak ***A***-fairness is *violated* if



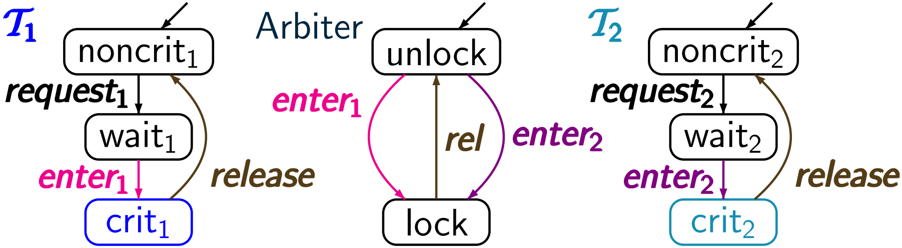
- no ***A***-actions are executed from a certain moment
- ***A***-actions are continuously enabled from some moment on

Mutual exclusion with arbiter

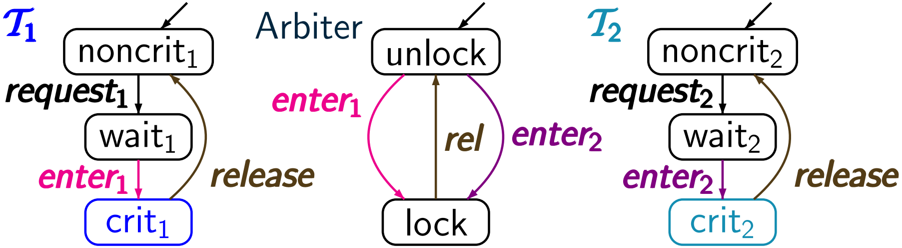




Mutual exclusion with arbiter



Mutual exclusion with arbiter



$\mathcal{T}_1$

noncrit$_1$

request$_1$

wait$_1$

enter$_1$

crit$_1$

release

Arbiter

unlock

enter$_1$

rel

enter$_2$

lock

$\mathcal{T}_2$

noncrit$_2$

request$_2$

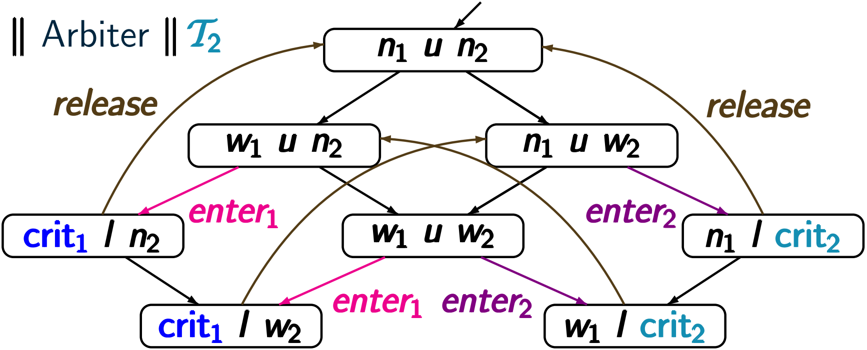
wait$_2$

enter$_2$

crit$_2$

release

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



n$_1$ u n$_2$

release

release

w$_1$ u n$_2$

n$_1$ u w$_2$

enter$_1$

enter$_2$

crit$_1$ *l n$_2$*

w$_1$ u w$_2$

n$_1$ l **crit$_2$**

crit$_1$ *l w$_2$*

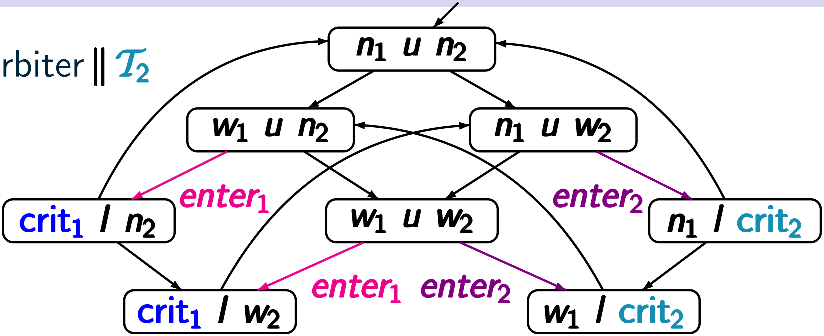
enter$_1$

enter$_2$

w$_1$ l **crit$_2$**

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



Unconditional, strongly or weakly fair?

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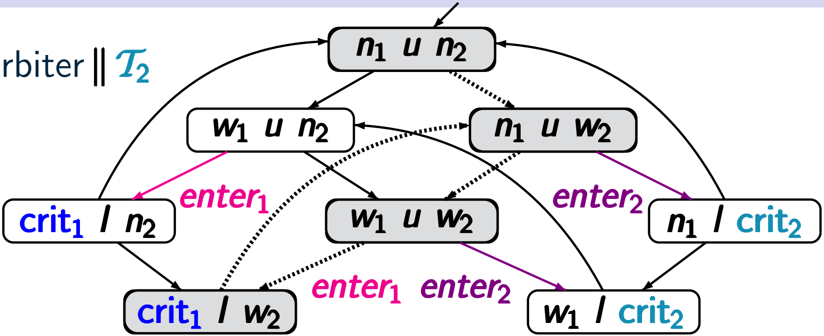


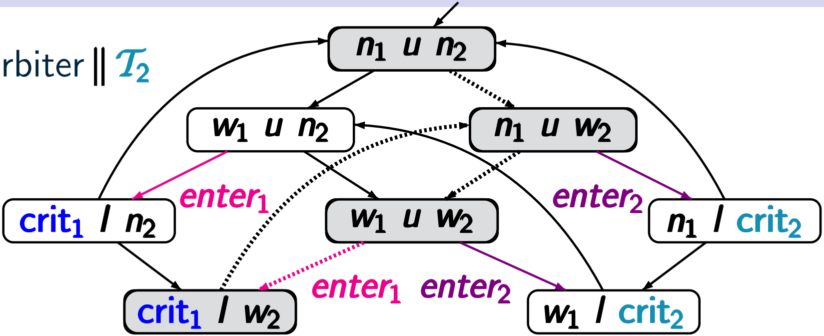
fairness for action set $A = \{enter_1\}$:

$$\langle n_1, u, n_2\rangle \rightarrow \Big(\langle n_1, u, w_2\rangle \rightarrow \langle w_1, u, w_2\rangle \rightarrow \langle crit_1, l, w_2\rangle\Big)^{\omega}$$

- unconditional A-fairness:
- strong A-fairness:
- weak A-fairness:

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



fairness for action set $A = \{enter_1\}$:

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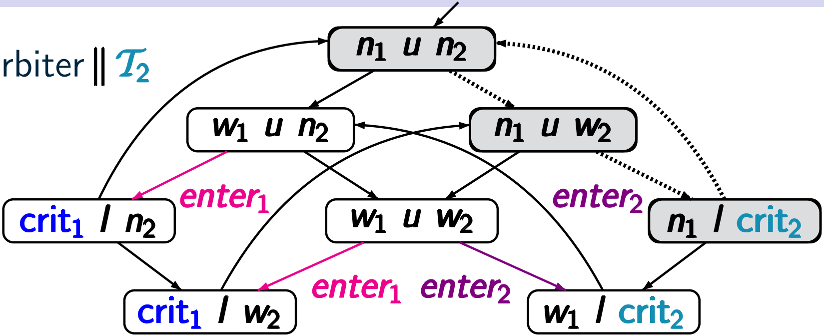
- unconditional A-fairness: **yes**
- strong A-fairness: **yes** ← unconditionally fair
- weak A-fairness: **yes** ← unconditionally fair

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



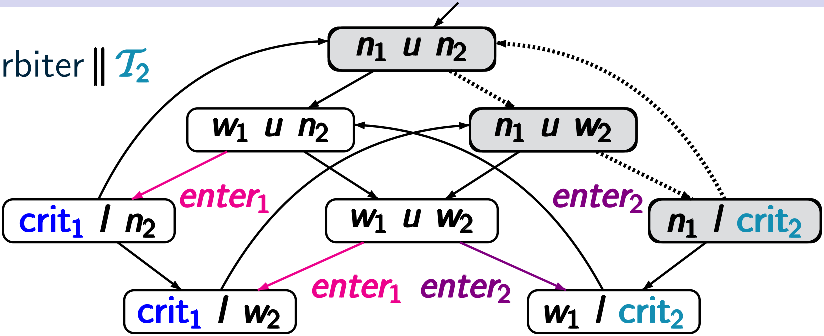
fairness for action-set $A = \{enter_1\}$:

$$\Big(\langle n_1, u, n_2\rangle \longrightarrow \langle n_1, u, w_2\rangle \longrightarrow \langle n_1, l, crit_2\rangle\Big)^{\omega}$$

- unconditional A-fairness:
- strong A-fairness:
- weak A-fairness:

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



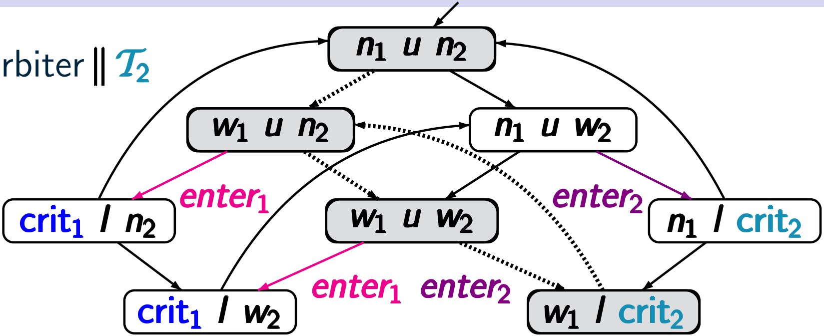
fairness for action-set $A = \{enter_1\}$:

$$\Big(\langle n_1, u, n_2\rangle \longrightarrow \langle n_1, u, w_2\rangle \longrightarrow \langle n_1, l, crit_2\rangle\Big)^{\omega}$$

- unconditional A-fairness: **no**
- strong A-fairness: **yes** ← A never enabled
- weak A-fairness: **yes** ← strongly A-fair

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



fairness for action-set $A = \{enter_1\}$:

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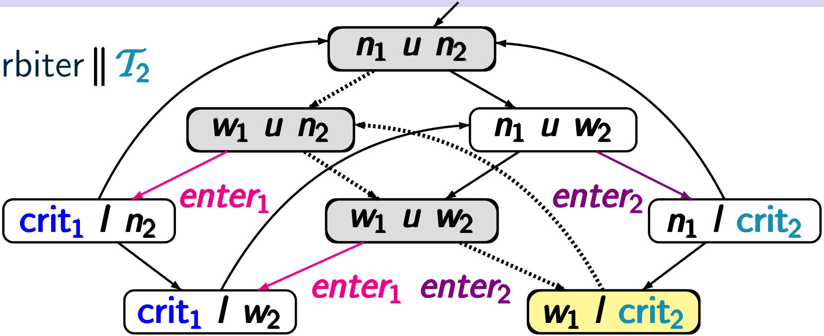
- unconditional A-fairness:
- strong A-fairness:
- weak A-fairness:

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



fairness for action-set $A = \{enter_1\}$:

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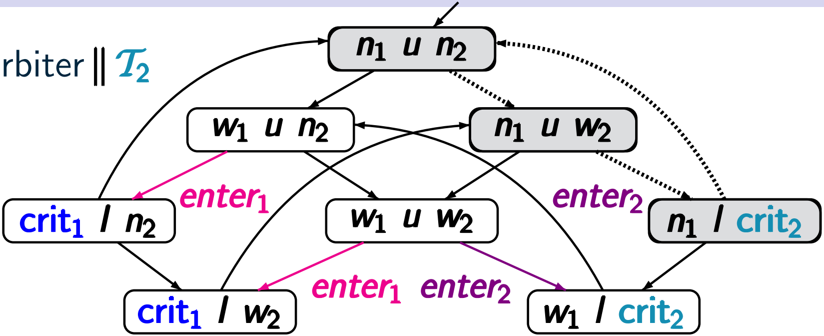
- unconditional A-fairness: **no**
- strong A-fairness: **no**
- weak A-fairness: **yes**

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



fairness for action set $A = \{enter_1, enter_2\}$:

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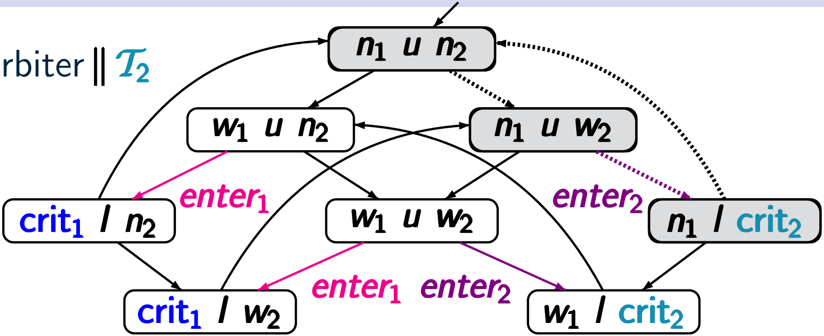
- unconditional A-fairness:
- strong A-fairness:
- weak A-fairness:

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



fairness for action set $A = \{enter_1, enter_2\}$:

$$\Big(\langle n_1, u, n_2 \rangle \rightarrow \langle n_1, u, w_2 \rangle \rightarrow \langle n_1, u, crit_2 \rangle \Big)^{\omega}$$

- unconditional A-fairness: **yes**
- strong A-fairness: **yes**
- weak A-fairness: **yes**

Action-based fairness assumptions

LF2.6-DEF-FAIRNESS-ASSUMPTION

Action-based fairness assumptions

Let $\mathcal{T}$ be a transition system with action-set Act.
A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$.

Action-based fairness assumptions

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where $\mathcal{F}_{ucond}, \; \mathcal{F}_{strong}, \; \mathcal{F}_{weak} \subseteq 2^{Act}$.

An execution ρ is called $\mathcal{F}$-fair iff

- ρ is unconditionally A-fair for all $A \in \mathcal{F}_{ucond}$
- ρ is strongly A-fair for all $A \in \mathcal{F}_{strong}$
- ρ is weakly A-fair for all $A \in \mathcal{F}_{weak}$

Action-based fairness assumptions

Let $\mathcal{T}$ be a transition system with action-set Act. A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

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- ρ is weakly A-fair for all $A \in \mathcal{F}_{weak}$

$$FairTraces_{\mathcal{F}}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\rho) : \rho \text{ is a } \mathcal{F}\text{-fair execution of } \mathcal{T}\}$$

Fair satisfaction relation

LF2.6-FAIR-SAT

Fair satisfaction relation

A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak} \subseteq 2^{Act}$.

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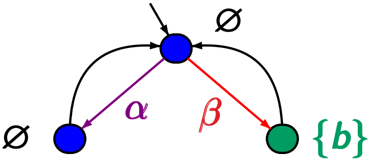
- ρ is strongly A-fair for all $A \in \mathcal{F}_{strong}$
- ρ is weakly A-fair for all $A \in \mathcal{F}_{weak}$

If $\mathcal{T}$ is a TS and E a LT property over AP then:

$$\mathcal{T} \models_{\mathcal{F}} E \quad \overset{\text{def}}{\iff} \quad FairTraces_{\mathcal{F}}(\mathcal{T}) \subseteq E$$

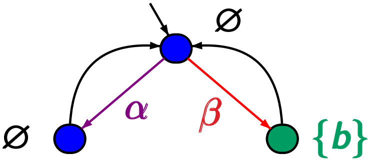
Example: fair satisfaction relation



fairness assumption $\mathcal{F}$

- no unconditional fairness condition
- strong fairness for $\{\alpha, \beta\}$
- no weak fairness condition

Example: fair satisfaction relation



fairness assumption $\mathcal{F}$

- no unconditional fairness condition ← $\mathcal{F}_{ucond} = \varnothing$
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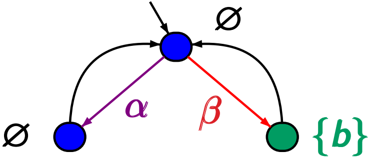

$\mathcal{T} \models_{\mathcal{F}}$ "infinitely often b" ?

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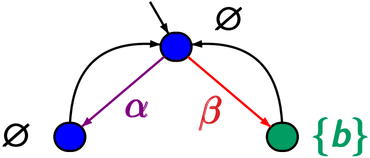

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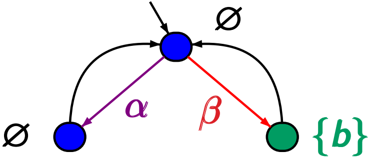

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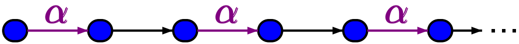
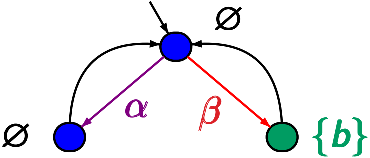


$\mathcal{F}$-fair

actions in $\{\alpha, \beta\}$ are executed infinitely many times

Example: fair satisfaction relation



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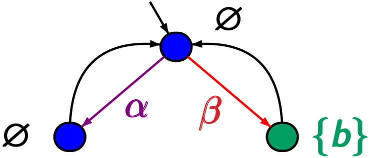

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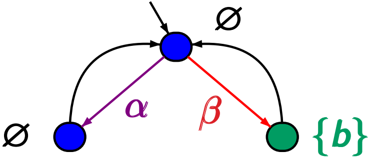

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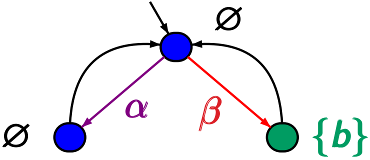

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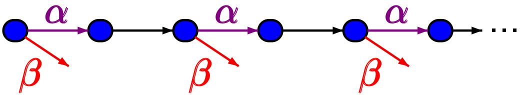
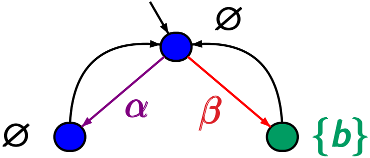


$\mathcal{F}$-fair

Example: fair satisfaction relation



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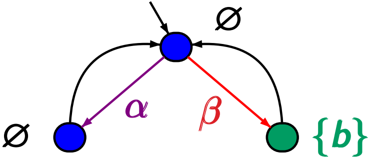

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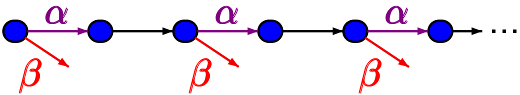


is not $\mathcal{F}$-fair

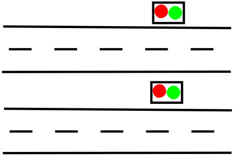
Which type of fairness?

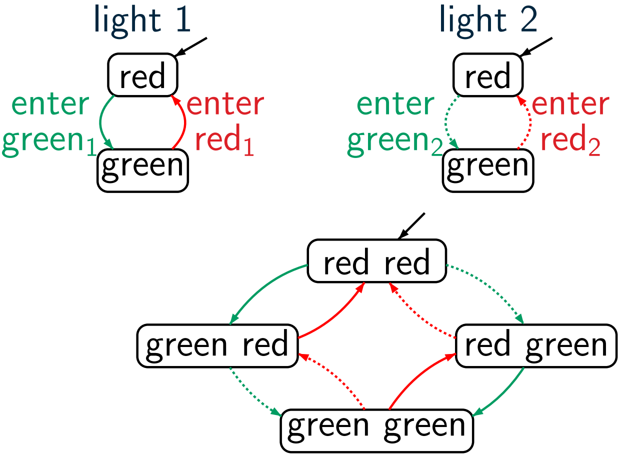
LF2.6-13A

Which type of fairness?

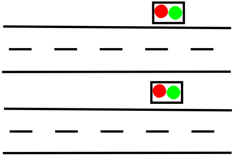
fairness assumptions should be
as weak as possible

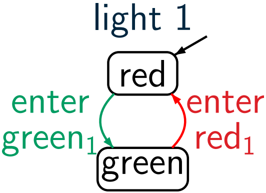
Two independent traffic lights

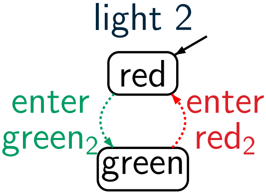


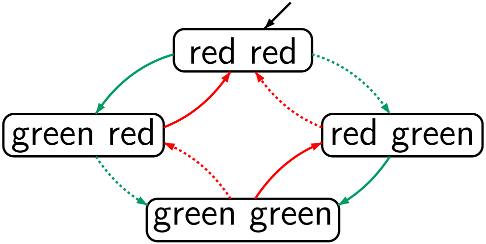


Two independent traffic lights









fairness assumption $\mathcal{F}$:

$\mathcal{F}_{ucond} = ?$

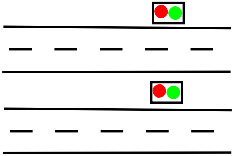
$\mathcal{F}_{strong} = ?$

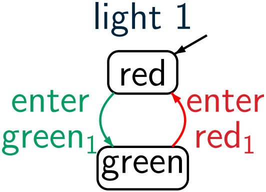
$\mathcal{F}_{weak} = ?$

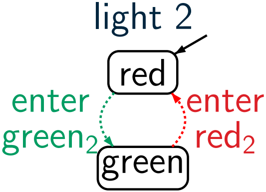
light 1 $|||$ light 2 $\models_{\mathcal{F}} E$

$E \mathrel{\widehat{=}}$ "both lights are infinitely often green"

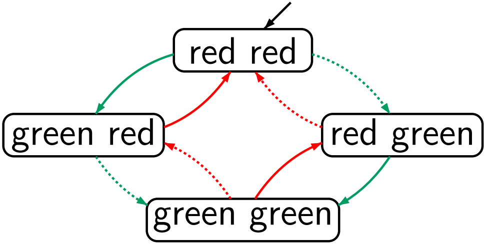
Two independent traffic lights







A_1 = actions of light 1
A_2 = actions of light 2



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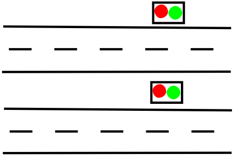
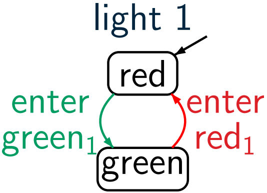




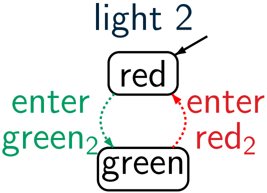
light 1

red

enter green$_1$

enter red$_1$

green



light 2

red

enter green$_2$

enter red$_2$

green

A_1 = actions of light 1

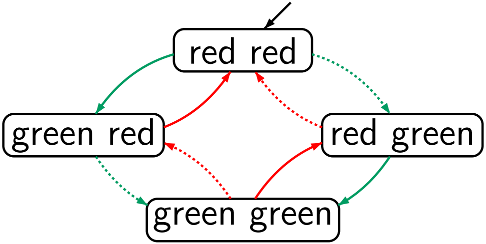
A_2 = actions of light 2

fairness assumption $\mathcal{F}$:

$\mathcal{F}_{ucond} = \varnothing$

$\mathcal{F}_{strong} = \varnothing$

$\mathcal{F}_{weak} = \{A_1, A_2\}$



red red

green red

red green

green green

light 1 $|||$ light 2 $\models_{\mathcal{F}} E$

$E \mathrel{\widehat{=}}$ “both lights are infinitely often green”

Example: MUTEX with fair arbiter

$$\mathcal{T} = \mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$$

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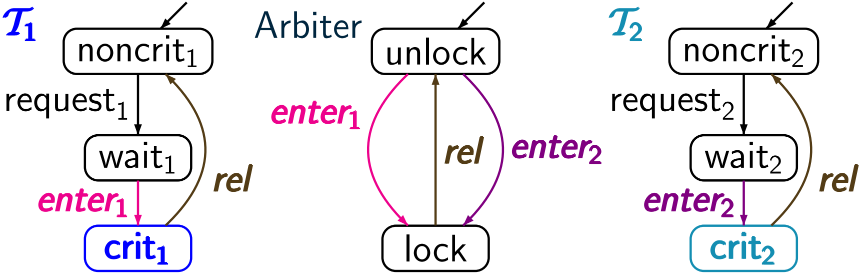
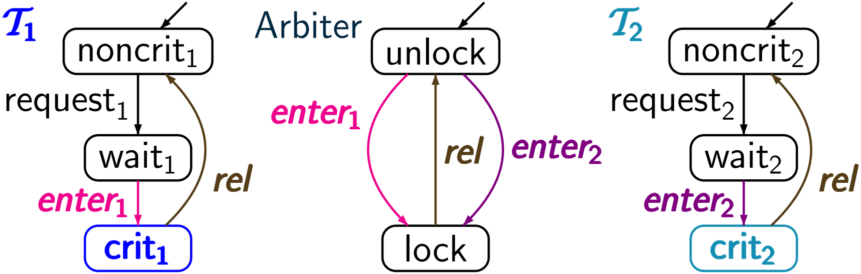


Example: MUTEX with fair arbiter

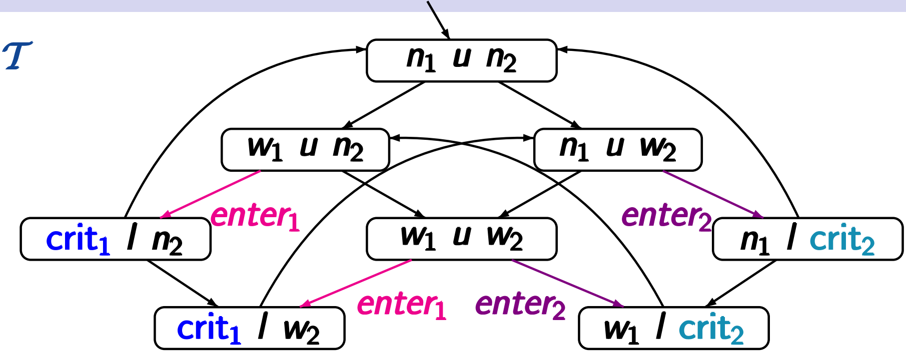
$\mathcal{T} = \mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



$\mathcal{T}_1$ and $\mathcal{T}_2$ compete to communicate with the arbiter by means of the actions ***enter***$_1$ and ***enter***$_2$, respectively

Example: MUTEX with fair arbiter

$\mathcal{T}$

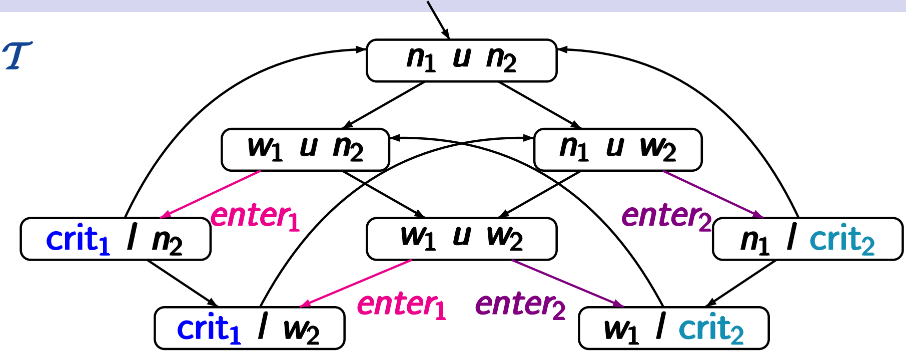


LT property E: each waiting process eventually enters its critical section

$\mathcal{T} \not\models E$

Example: MUTEX with fair arbiter

$\mathcal{T}$



LT property E: each waiting process eventually enters its critical section

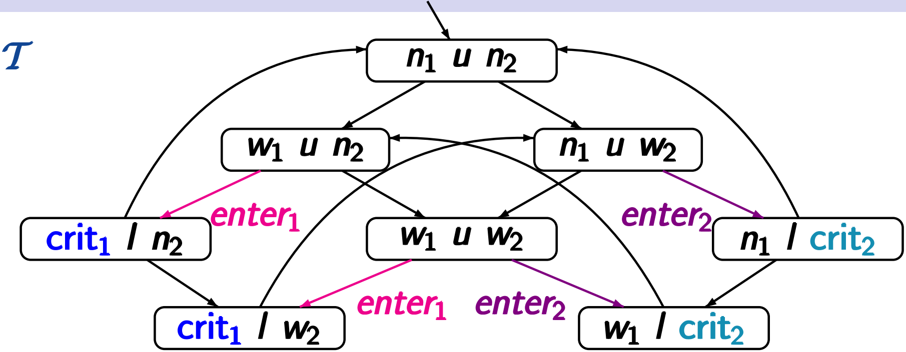
fairness assumption $\mathcal{F}$

$\mathcal{F}_{ucond} = \mathcal{F}_{strong} = \varnothing$

$\mathcal{F}_{weak} = \{\{enter_1\}, \{enter_2\}\}$

does $\mathcal{T} \models_{\mathcal{F}} E$ hold ?

Example: MUTEX with fair arbiter



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answer: **no**

Example: MUTEX with fair arbiter

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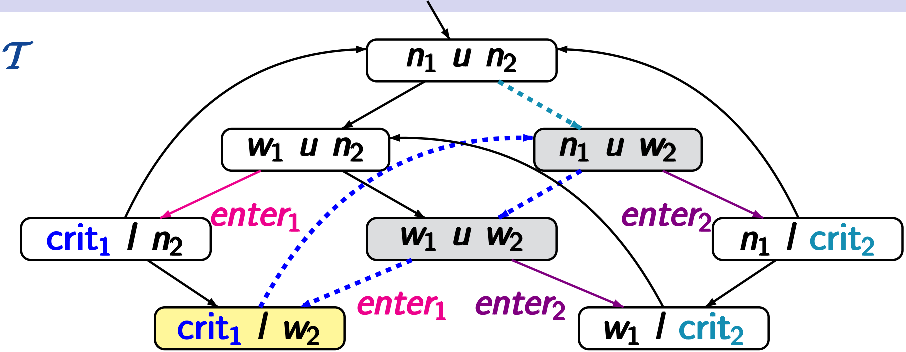

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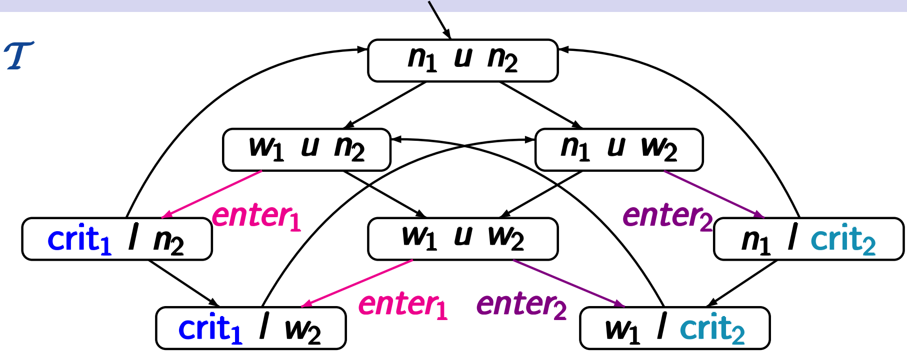
$\mathcal{T} \not\models_{\mathcal{F}} E$

as $enter_2$ is not enabled in $\langle crit_1, l, w_2 \rangle$

Example: MUTEX with fair arbiter



E: each waiting process eventually enters its crit. section

$\mathcal{F}_{ucond} = ?$
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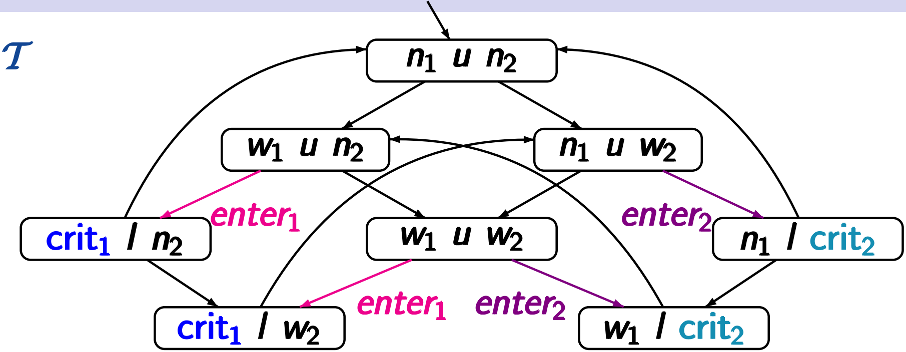
but $\mathcal{T} \models_{\mathcal{F}} E$

Example: MUTEX with fair arbiter

$\mathcal{T}$



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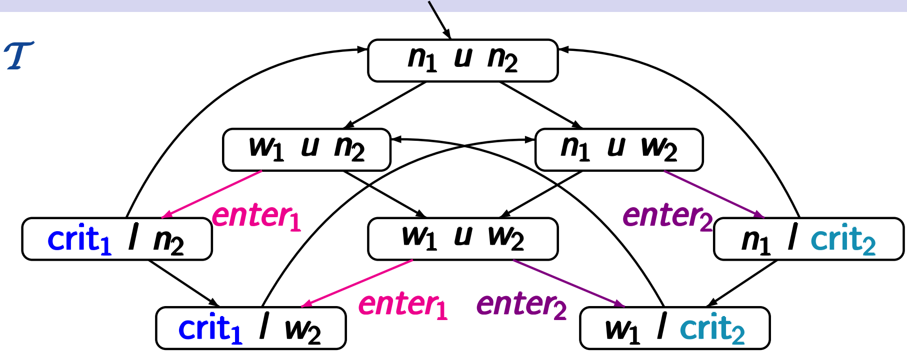


E: each waiting process eventually enters its crit. section

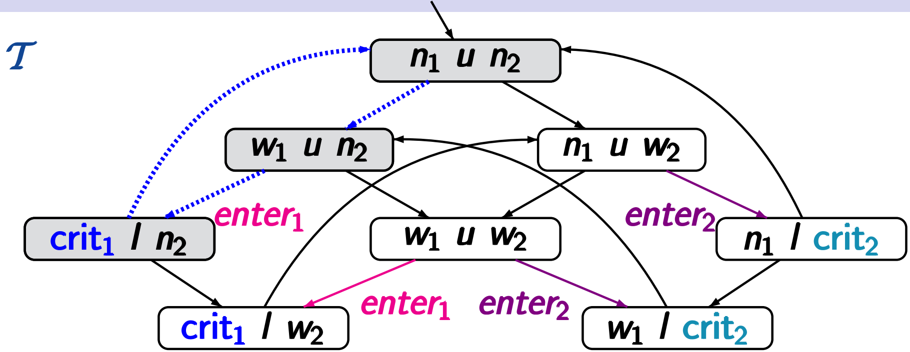
D: each process enters its critical section infinitely often

$$\begin{aligned} \mathcal{F}_{ucond} &= \varnothing \\ \mathcal{F}_{strong} &= \{\{enter_1\}, \{enter_2\}\} \\ \mathcal{F}_{weak} &= \varnothing \end{aligned}$$

$$\mathcal{T} \models_{\mathcal{F}} E, \qquad \mathcal{T} \not\models_{\mathcal{F}} D$$

Example: MUTEX with fair arbiter

$\mathcal{T}$



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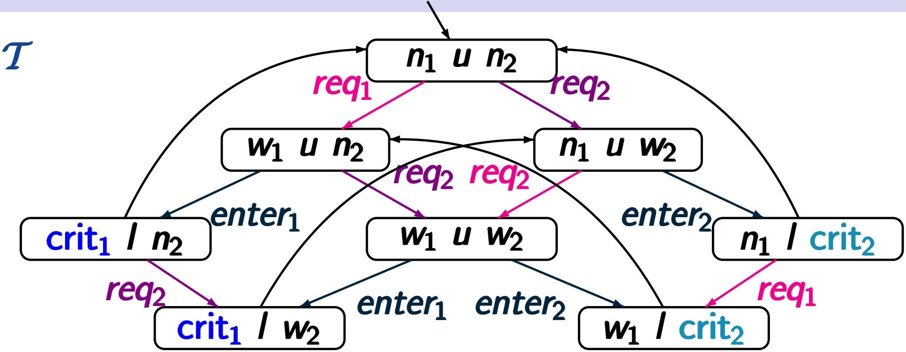

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$$\boxed{\begin{array}{l}\mathcal{T} \models_{\mathcal{F}} E, \\ \mathcal{T} \models_{\mathcal{F}} D\end{array}}$$

Process fairness

LF2.6-19

Process fairness

For asynchronous systems:

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

Process fairness

For asynchronous systems:

parallelism = interleaving + fairness

↑

should be as weak as possible

Process fairness

For asynchronous systems:

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \underset{\substack{\uparrow\\ \text{should be as weak as possible}}}{\text{fairness}}}$$

rule of thumb:

- strong fairness for the
  - ∗ choice between dependent actions
  - ∗ resolution of competitions

Process fairness

For asynchronous systems:

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

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- weak fairness for the nondetermism obtained from the interleaving of independent actions

Process fairness

For asynchronous systems:

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

↑ should be as weak as possible

rule of thumb:

- strong fairness for the
  - choice between dependent actions
  - resolution of competitions
- weak fairness for the nondetermism obtained from the interleaving of independent actions
- unconditional fairness: only of theoretical interest

Purpose of fairness conditions

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

Process fairness and other fairness conditions

- can compensate information loss due to interleaving
  or rule out other unrealistic pathological cases
- can be requirements for a scheduler
  or requirements for environment
- can be verifiable system properties

Purpose of fairness conditions

parallelism = interleaving + fairness

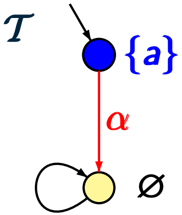
Process fairness and other fairness conditions

- can compensate information loss due to interleaving or rule out other unrealistic pathological cases
- can be requirements for a scheduler or requirements for environment
- can be verifiable system properties

liveness properties: fairness can be essential

safety properties: fairness is irrelevant

Fairness

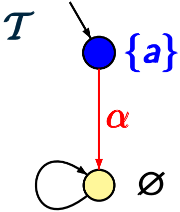


fairness assumption $\mathcal{F}$:

unconditional fairness
for action set $\{\alpha\}$

does $\mathcal{T} \models_{\mathcal{F}}$ "infinitely often a" hold ?

Fairness



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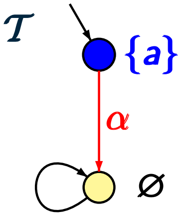
answer: **yes** as there is no fair path

Fairness



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for action set $\{\alpha\}$

not realizable

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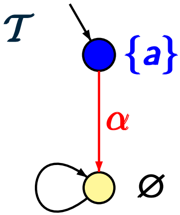
Realizability of fairness assumptions



fairness assumption $\mathcal{F}$:
unconditional fairness
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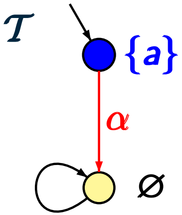
↑
not realizable

does $\mathcal{T} \models_{\mathcal{F}}$ "infinitely often a" hold **?**

answer: **yes** as there is no fair path

Realizability requires that each initial finite path fragment can be extended to a $\mathcal{F}$-fair path

Realizability of fairness assumptions



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Fairness assumption $\mathcal{F}$ is said to be realizable for a transition system $\mathcal{T}$ if for each reachable state s in $\mathcal{T}$ there exists a $\mathcal{F}$-fair path starting in s

Realizability of fairness assumptions

LF2.6-23

Realizability of fairness assumptions

fairness assumption $\mathcal{F} = (\mathcal{F}_{\mathit{ucond}}, \mathcal{F}_{\mathit{strong}}, \mathcal{F}_{\mathit{weak}})$ for TS $\mathcal{T}$

Realizability of fairness assumptions

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↑

can always be guaranteed by a scheduler, i.e.,
an instance that resolves the nondeterminism in $\mathcal{T}$

Safety and realizable fairness

LF2.6-24

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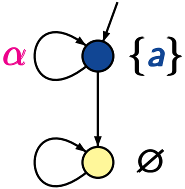

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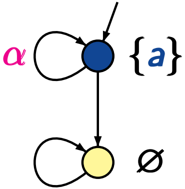

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E = invariant "always a"

$\mathcal{T} \not\models E$, but $\mathcal{T} \models_{\mathcal{F}} E$