Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

- regular safety properties
- ω-regular properties
- model checking with Büchi automata $\longleftarrow$

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Verifying ω-regular properties

given: finite transition system $\mathcal{T}$
ω-regular property E

question: does $\mathcal{T} \models E$ hold ?

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requires techniques for checking
persistence properties in finite TS

Persistence property

Let E be an LT-property, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$

> E is called a persistence property if there exists a propositional formula Φ over AP such that
>
> $$E = \left\{ \begin{array}{l} \text{set of all infinite words } A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega} \\ \qquad\qquad \text{s.t.} \quad \overset{\infty}{\forall}\, i \geq 0.\ A_i \models \Phi \end{array} \right.$$

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"from some moment on Φ"
"eventually forever Φ"

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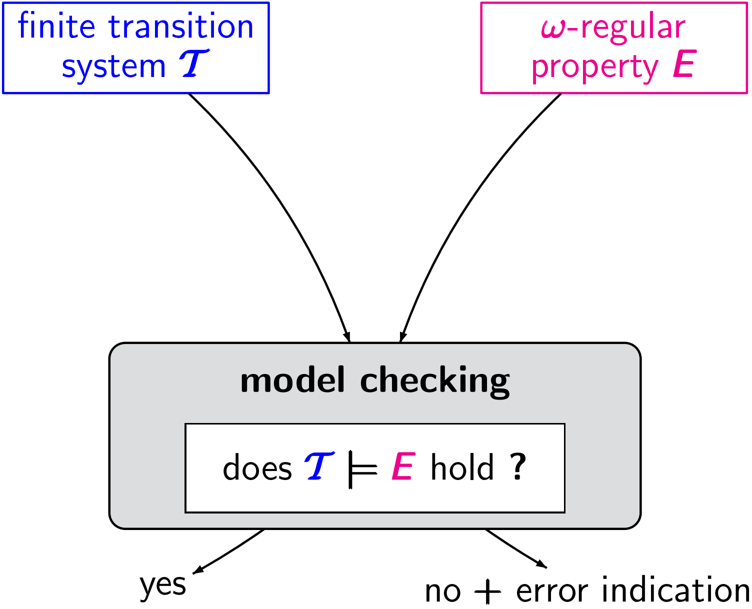
Checking ω-regular properties



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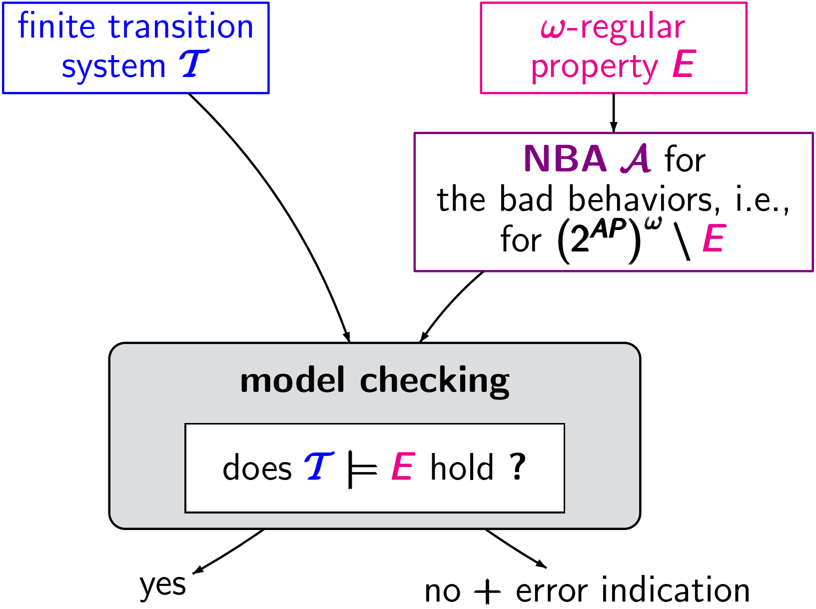

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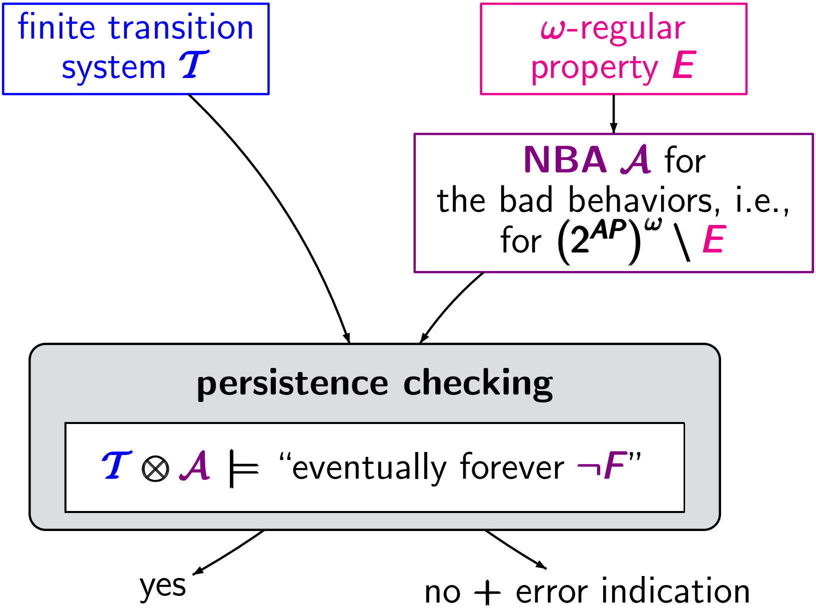

Recall: product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \to, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

$$\begin{array}{c} s_0 \\ \downarrow \\ s_1 \\ \downarrow \\ s_2 \\ \downarrow \\ \vdots \\ \downarrow \\ s_n \end{array}$$

path
fragment $\widehat{\pi}$

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$$\begin{array}{ccc}
s_0 & & L(s_0)=A_0 \\
\downarrow & & \\
s_1 & & L(s_1)=A_1 \\
\downarrow & & \\
s_2 & & L(s_2)=A_2 \\
\downarrow & & \\
\vdots & & \vdots \\
\downarrow & & \\
s_n & & L(s_n)=A_n
\end{array}$$

path fragment $\widehat{\pi}$ | trace

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$$\begin{array}{ccc}
s_0 & L(s_0)=A_0 & q_0 \in Q_0 \\
\downarrow & & \downarrow A_0 \\
s_1 & L(s_1)=A_1 & q_1 \\
\downarrow & & \downarrow A_1 \\
s_2 & L(s_2)=A_2 & q_2 \\
\downarrow & & \downarrow A_2 \\
\vdots & \vdots & \vdots \\
\downarrow & & \downarrow \\
s_n & L(s_n)=A_n & q_n \\
 & & \downarrow A_n \\
 & & q_{n+1}
\end{array}$$

path fragment $\widehat{\pi}$ | trace | run for $trace(\widehat{\pi})$

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$$q_0 \in Q_0$$

path fragment $\widehat{\pi}$	trace	path fragm. in product	run for $trace(\widehat{\pi})$
s_0	$L(s_0) = A_0$	$\langle s_0, q_1 \rangle$	$q_0 \xrightarrow{A_0} q_1$
s_1	$L(s_1) = A_1$	$\langle s_1, q_2 \rangle$	$q_1 \xrightarrow{A_1} q_2$
s_2	$L(s_2) = A_2$	$\langle s_2, q_3 \rangle$	$q_2 \xrightarrow{A_2} \ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_n	$L(s_n) = A_n$	$\langle s_n, q_{n+1} \rangle$	$q_n \xrightarrow{A_n} q_{n+1}$

Product transition system

recall: definition of the product of a **TS** and **NFA**

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

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$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

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set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA ← same definition for **NBA**

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$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$

$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F) \quad \textbf{NFA} \text{ or } \textbf{NBA}$$

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S'_0, AP', L')$

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ω-regular model checking

given:	finite TS $\mathcal{T}$
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relies on a reduction to the persistence checking problem

ω-regular model checking

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ finite transition system without terminal states

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representing the bad behaviors of an ω-regular LT-property E

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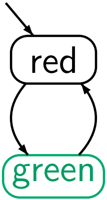
Example: ω-regular model checking

TS $\mathcal{T}$



LT property: "infinitely often green"

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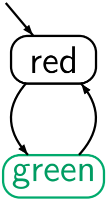

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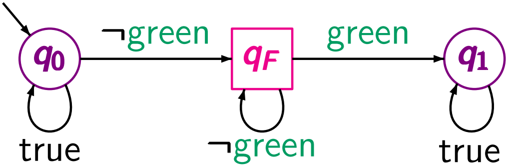
NBA $\mathcal{A}$ for the complement

"from some moment on ¬green"



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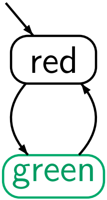

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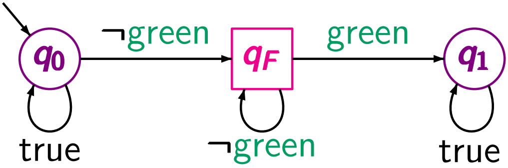

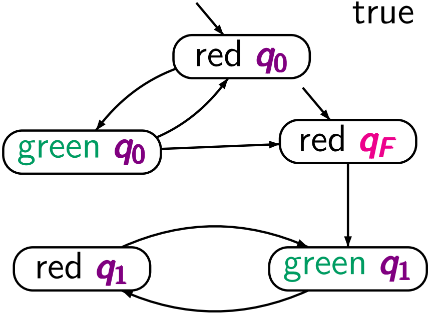

reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

Example: ω-regular model checking

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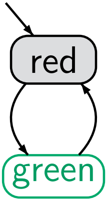

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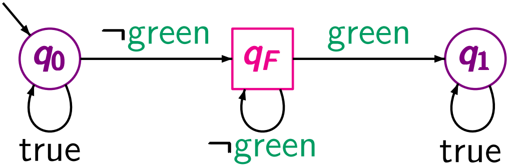

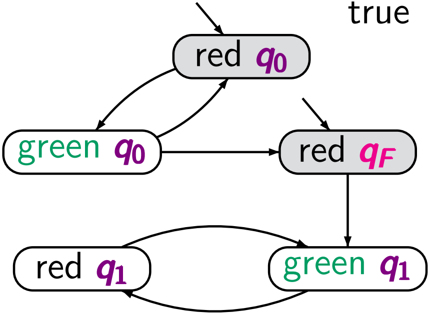

initial states:

$\langle \text{red}, q \rangle$ where

$$
\begin{aligned}
q &\in \delta(q_0, L(\text{red})) \\
&= \delta(q_0, \varnothing) \\
&= \{q_0, q_F\}
\end{aligned}
$$

Example: ω-regular model checking

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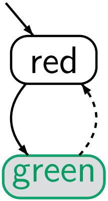

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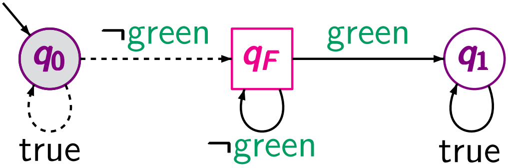

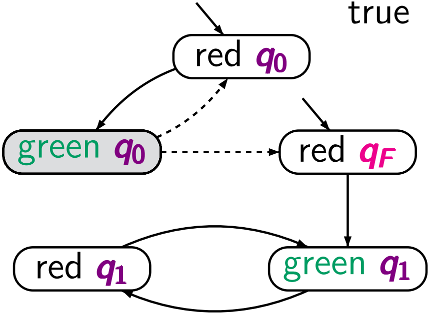

transition

$$\langle \text{green}, q_0 \rangle \to \langle \text{red}, q \rangle$$

$$\begin{aligned} q &\in \delta(q_0, L(\text{red})) \\ &= \delta(q_0, \varnothing) \\ &= \{q_0, q_F\} \end{aligned}$$

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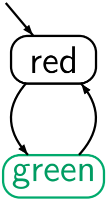

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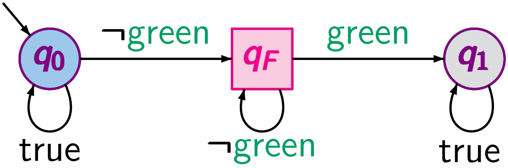
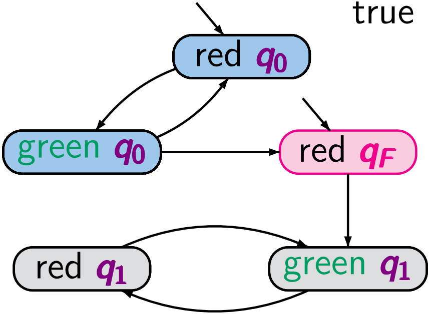




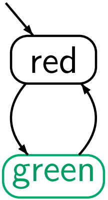
atomic propositions

$AP' = \{q_0, q_F, q_1\}$

obvious labeling function

Example: ω-regular model checking

TS $\mathcal{T}$



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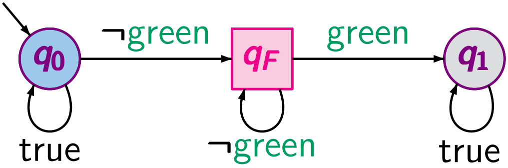

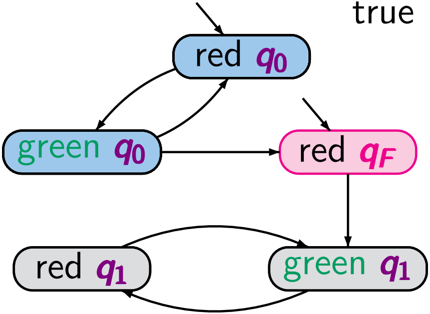

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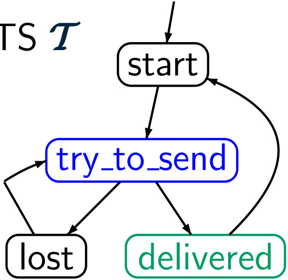
TS $\mathcal{T}$

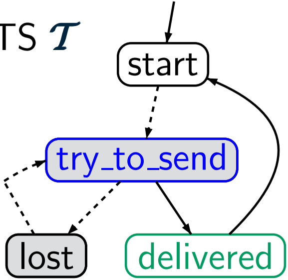


ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

Example: ω-regular model checking

TS $\mathcal{T}$



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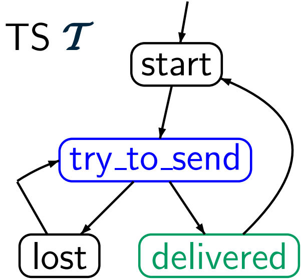
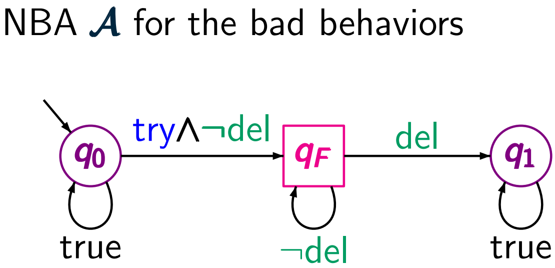
$\mathcal{T} \not\models E$

Example: ω-regular model checking





ω-regular LT property E:

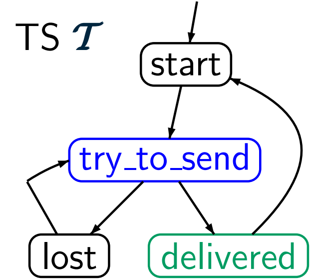
"each (repeatedly) sent message will eventually be delivered"

complement of E, i.e., LT property for the bad behaviors:

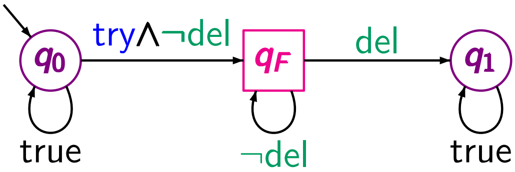
"never delivered after some trial"

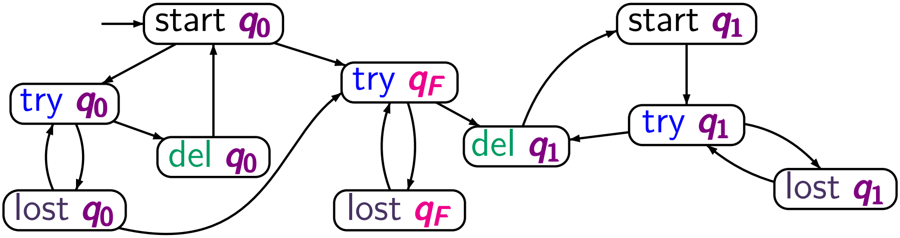
Example: ω-regular model checking

TS $\mathcal{T}$



NBA $\mathcal{A}$ for the bad behaviors

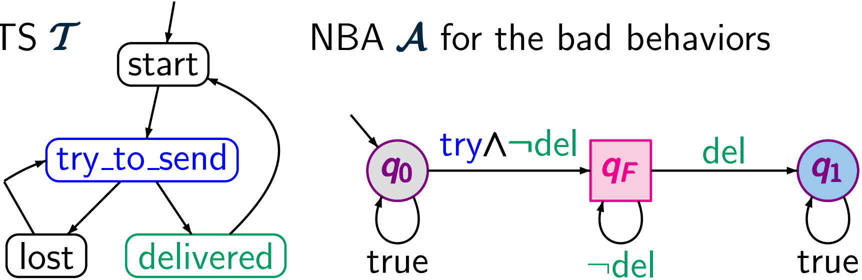


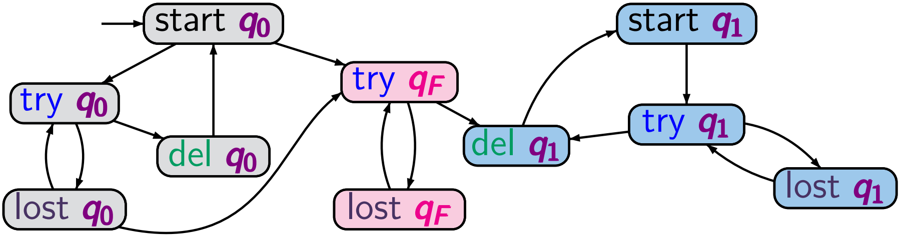


reachable fragment of the product-TS

Example: ω-regular model checking

TS $\mathcal{T}$ NBA $\mathcal{A}$ for the bad behaviors

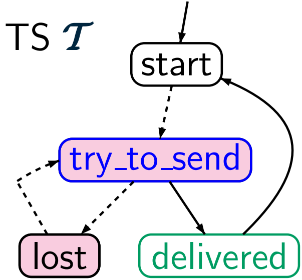




set of atomic propositions $AP' = \{q_0, q_1, q_F\}$

Example: ω-regular model checking

TS $\mathcal{T}$



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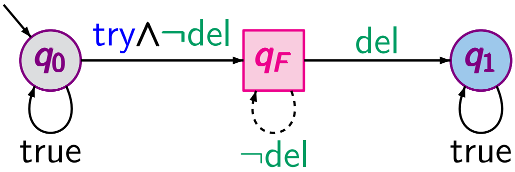

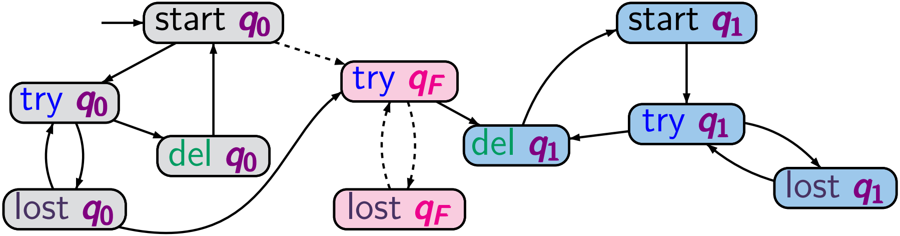

$\mathcal{T} \otimes \mathcal{A} \not\models$ "eventually forever $\neg F$"

Checking safety and ω-regular properties

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F = set of final states in $\mathcal{A}$

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A SCC is called non-trivial if it has at least one edge.
"either 1 state with a self-loop or 2 or more states"

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method: calculate and analyze the **SCCs**

Example: ω-regular model checking

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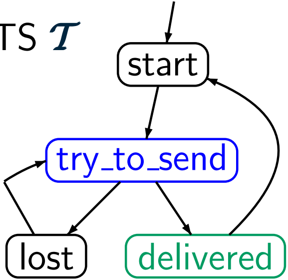

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"each (repeatedly) sent message will eventually be delivered"

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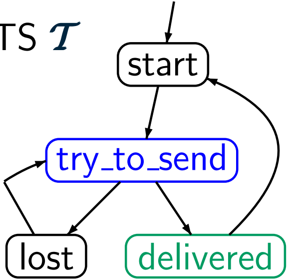
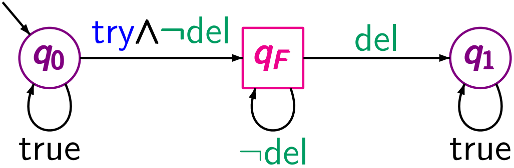


NBA $\mathcal{A}$ for the bad behaviors



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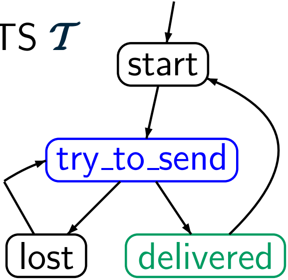

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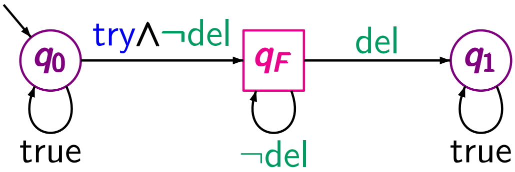

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... analysis of the **SCCs** in product $\mathcal{T} \otimes \mathcal{A}$...

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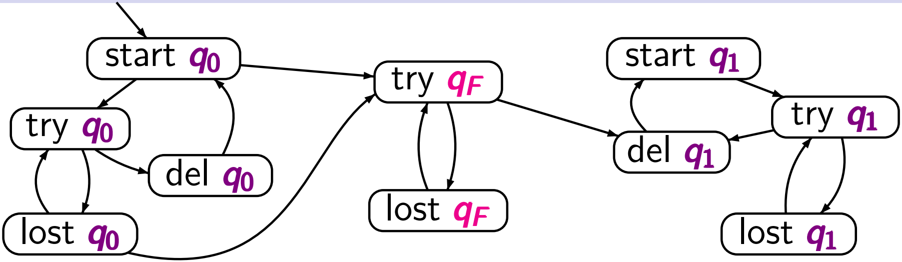

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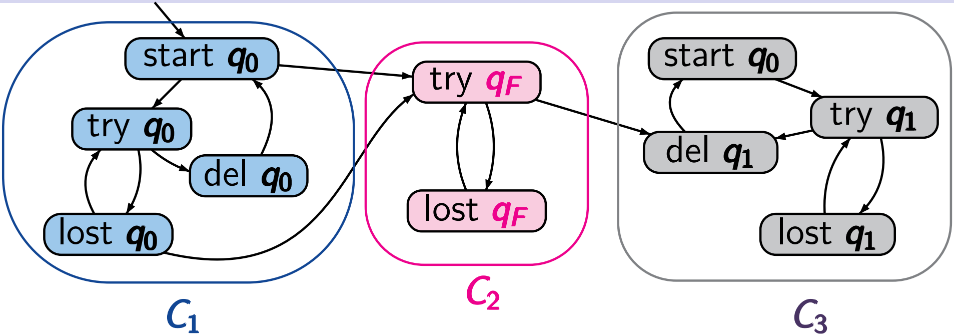

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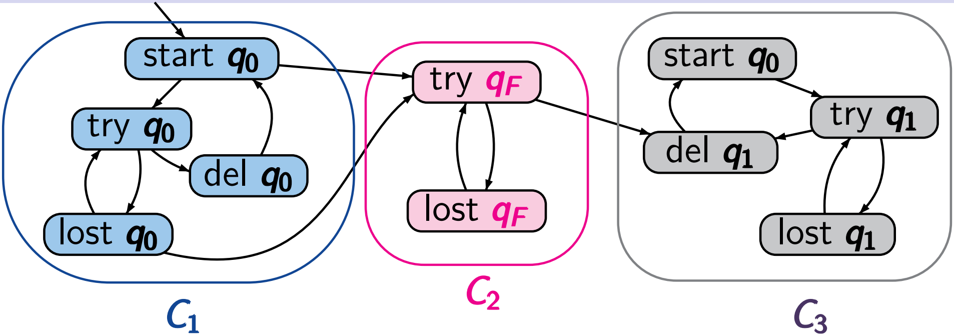

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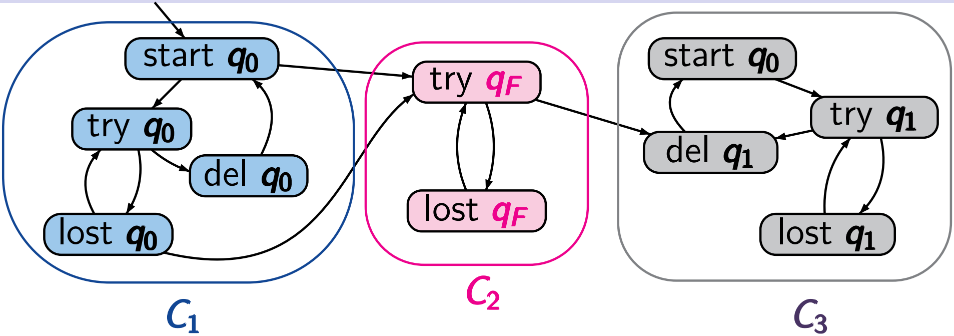

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Persistence checking algorithms based on ...

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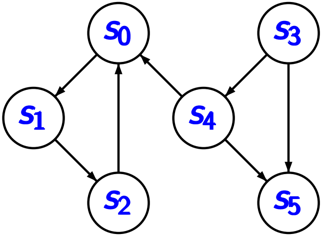
method 2: **DFS**-based search for **backward edges**

DFS forests and DFS edge-classification



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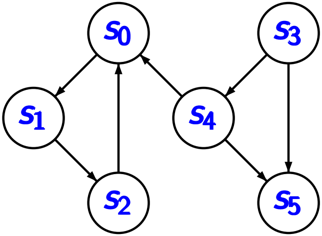

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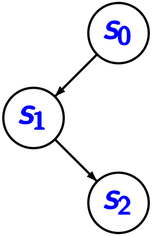

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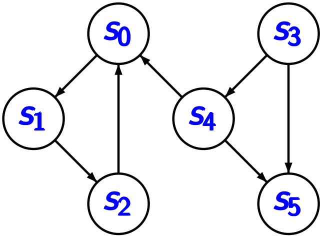

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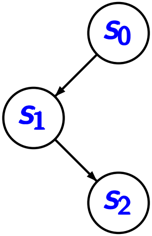

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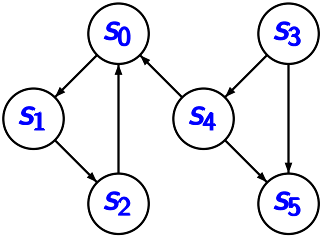

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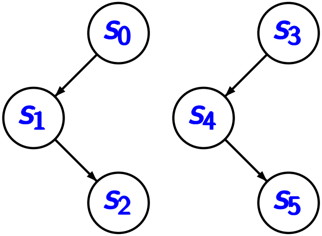

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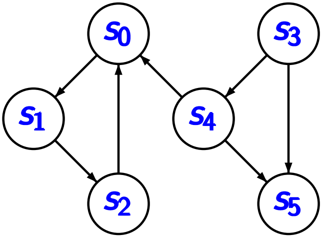

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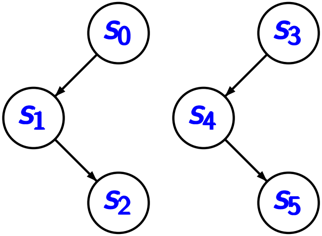

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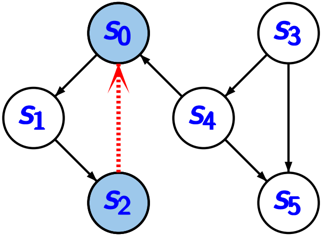
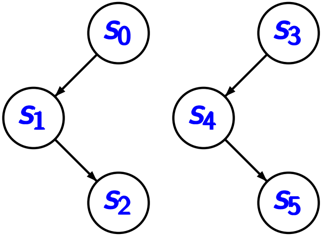


backward edge

$s_2 \longrightarrow s_0$

"closes" a cycle

DFS-forest, e.g.,



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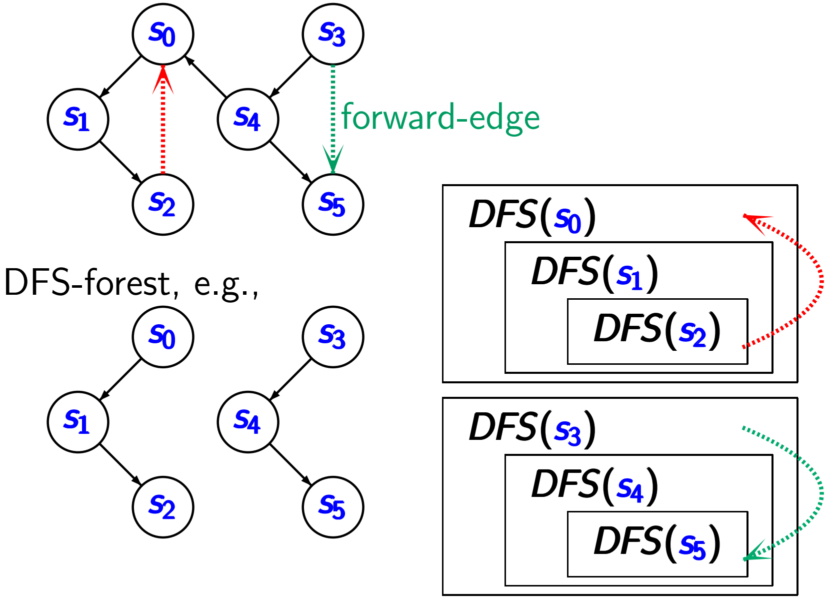

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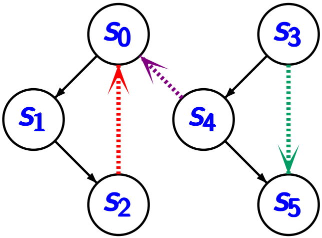
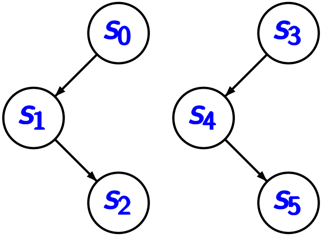


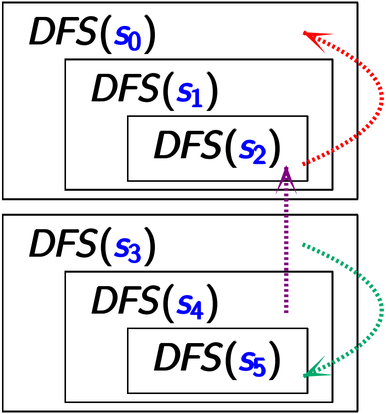
cross edge

$s_4 \longrightarrow s_0$

connects two (sub)trees

DFS-forest, e.g.,





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Let G be a finite directed graph and s a node in G.

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(1) s belongs to a cycle $s\, s_1\, s_2 \ldots s_k\, s$

(2) The DFS started with s finds a backward edge $s' \longrightarrow s$.

Cycle check for fixed node: "does s belong to a cycle?"

- perform by a DFS with starting node s
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initially all states are unmarked

REPEAT

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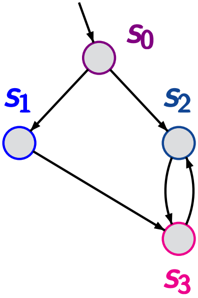
- uses a global visiting set V of states that have been visited so far in the **2. DFS**

Example: nested DFS

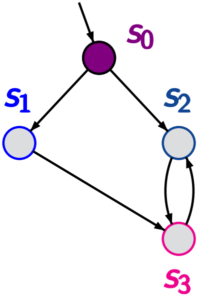


$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



$s_1, s_2 \not\models a$

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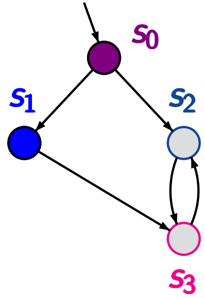
$DFS(s_0)$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



$s_1, s_2 \not\models a$

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$DFS(s_0)$

$DFS(s_1)$

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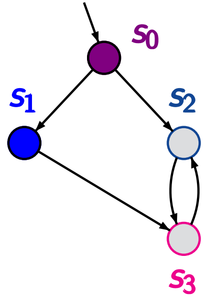

$s_1, s_2 \not\models a$

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$DFS(s_0)$

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$CYCLE\_CHECK(s_1)$

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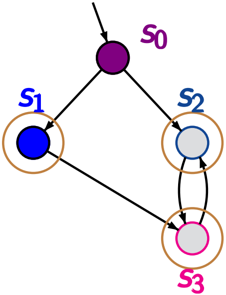
Example: nested DFS



$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

DFS(s_0)

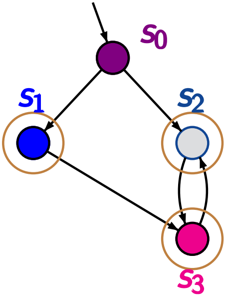
DFS(s_1)

CYCLE_CHECK(s_1)

$V = \{s_1, s_2, s_3\}$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



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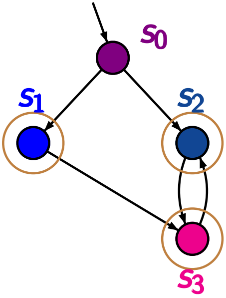
Example: nested DFS



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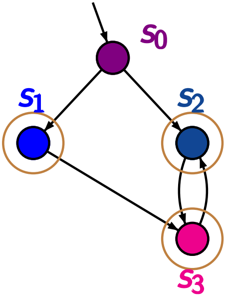

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$DFS(s_1)$

$CYCLE\_CHECK(s_1)$

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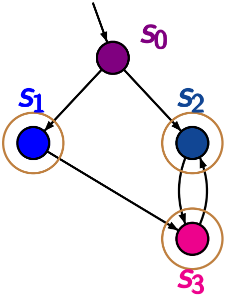
$DFS(s_3)$

$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS $\longleftarrow$ fails



$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

$DFS(s_0)$

$DFS(s_1)$

$CYCLE\_CHECK(s_1)$

$V = \{s_1, s_2, s_3\}$

$DFS(s_3)$

$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\uparrow$

returns wrong answer "**yes**"

$\mathcal{T} \not\models$ "eventually forever a"

- serves to check whether $\mathcal{T} \models$ "eventually forever a"
- relies on two DFSs running in an interleaved way

1. **DFS:** visits all reachable states
2. **DFS:** *CYCLE_CHECK*(s) for states s with $s \not\models a$

Nested DFS

- serves to check whether $\mathcal{T} \models$ "eventually forever a"
- relies on two DFSs running in an interleaved way

1. DFS: visits all reachable states

2. DFS: *CYCLE_CHECK*(s) for states s with $s \not\models a$

- checks whether s belongs to a cycle by searching a backward edge $s' \to s$
- ignores states that have been visited in previous calls of *CYCLE_CHECK* by using a global visiting set V for the **2. DFS**

Nested DFS

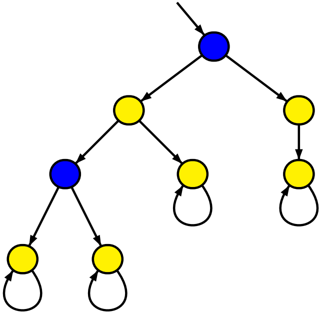
- serves to check whether $\mathcal{T} \models$ "eventually forever a"
- relies on two DFSs running in an interleaved way

1. DFS: visits all reachable states

2. DFS: *CYCLE_CHECK*(s) for states s with $s \not\models a$

- checks whether s belongs to a cycle
  by searching a backward edge $s' \rightarrow s$
- ignores states that have been visited in previous calls of *CYCLE_CHECK* by using a global visiting set V for the **2. DFS**
- is called for state s after s is **fully expanded** in the **1. DFS**

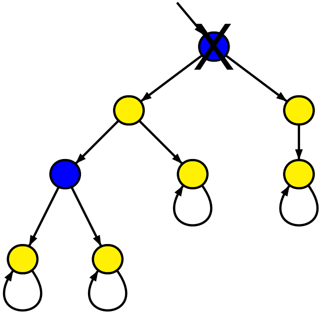
Nested DFS



$\mathcal{T} \models$ "eventually forever $\neg$***blue***"

1. **DFS:** visits all reachable states
2. **DFS:** *CYCLE_CHECK*(s) for $s \models$ ***blue***
   checks whether s belongs to a cycle
   by searching a backward-edge $s' \to s$

Nested DFS



$\mathcal{T} \models$ "eventually forever $\neg$***blue***"

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2. **DFS:** $\mathit{CYCLE\_CHECK}(s)$ for $s \models \mathit{blue}$
   checks whether s belongs to a cycle
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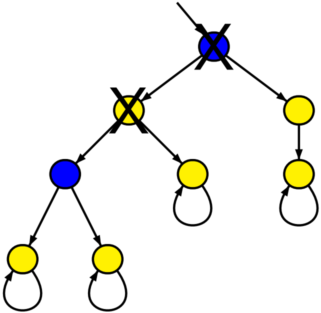
Nested DFS



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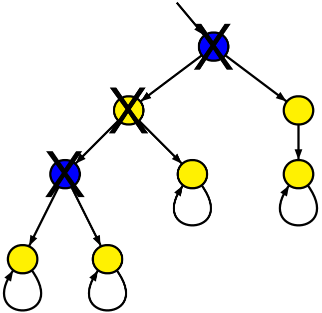

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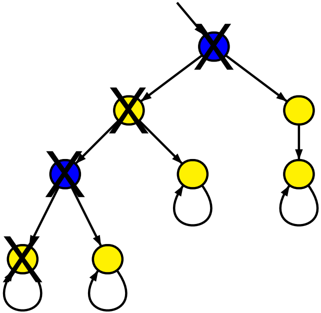
Nested DFS



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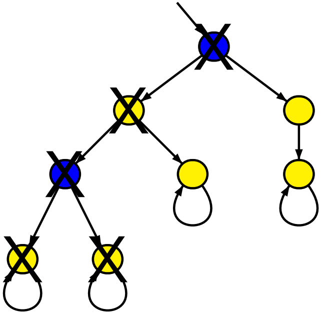
Nested DFS



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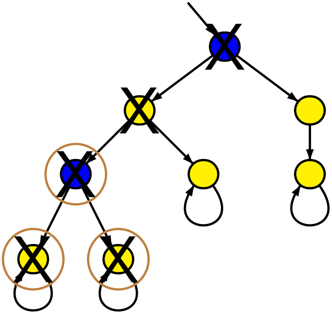

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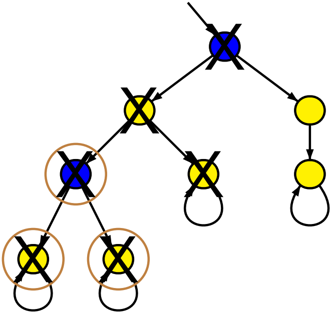

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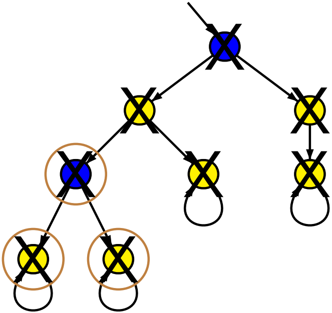
Nested DFS



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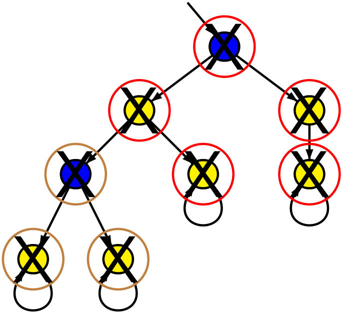
Nested DFS



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1. **DFS:** visits all reachable states
2. **DFS:** *CYCLE_CHECK*(s) for $s \models$ ***blue***
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Nested DFS (pseudo code)

$U := \varnothing;$

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

Nested DFS (pseudo code)

$U := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set of 1. DFS}}$

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

Nested DFS (pseudo code)

$U := \varnothing$; $\longleftarrow$ visiting set of 1. DFS

```
FOR ALL s₀ ∈ S₀ DO DFS(s₀) OD ;
```

```
IF s ∉ U THEN
    insert s in U;
```

pseudo code for $DFS(s)$

Nested DFS (pseudo code)

$U := \varnothing;$ $\longleftarrow$ visiting set of 1. DFS

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

pseudo code for $DFS(s)$

```
IF s ∉ U THEN
    insert s in U;

    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;
```

Nested DFS (pseudo code)

$U := \varnothing$; ⟵ visiting set of 1. DFS

```
FOR ALL s_0 ∈ S_0 DO DFS(s_0) OD ;
```

pseudo code for $DFS(s)$

```
IF s ∉ U THEN
    insert s in U;
    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;
    IF s ⊭ a THEN
        IF CYCLE_CHECK(s)
    FI
```

Nested DFS (pseudo code)

$U := \varnothing;$ ⟵ visiting set of 1. DFS

$V := \varnothing$ ⟵ global visiting set of 2. DFS

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

pseudo code for $DFS(s)$

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IF s ∉ U THEN
    insert s in U;
    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;
    IF s ⊭ a THEN
        IF CYCLE_CHECK(s)
    FI
```

Nested DFS (pseudo code)

$U := \varnothing$; ⟵ visiting set of 1. DFS

$V := \varnothing$ ⟵ global visiting set of 2. DFS

```
FOR ALL s0 ∈ S0 DO DFS(s0) OD ;
```

pseudo code for $DFS(s)$

```
IF s ∉ U THEN
    insert s in U;

    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;

    IF s ⊭ a THEN
        IF CYCLE_CHECK(s) THEN return "no" FI
FI
```

$U := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set of 1. DFS}}$

$V := \varnothing$ $\longleftarrow$ $\boxed{\text{global visiting set of 2. DFS}}$

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

$\boxed{\text{pseudo code for } DFS(s)}$

IF $s \notin U$ THEN

insert s in U;

FOR ALL $s' \in Post(s)$ DO $DFS(s')$ OD ;

IF $s \not\models a$ THEN

IF $CYCLE\_CHECK(s)$ THEN return "**no**" FI

FI

$\mathcal{T} \not\models$ "eventually forever a"

Nested DFS (pseudo code)

$U := \varnothing$; $\longleftarrow$ visiting set of 1. DFS

$V := \varnothing$ $\longleftarrow$ global visiting set of 2. DFS

```
FOR ALL s_0 ∈ S_0 DO DFS(s_0) OD ;
return "yes"
```

"yes" $\longleftarrow$ $\mathcal{T} \models$ "eventually forever a"

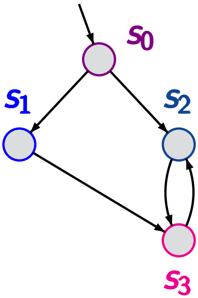
```
IF s ∉ U THEN
    insert s in U;

    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;

    IF s ⊭ a THEN
        IF CYCLE_CHECK(s) THEN return "no" FI
    FI
FI
```

pseudo code for $DFS(s)$

Example: nested DFS



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$\mathcal{T} \not\models$ "eventually forever a"

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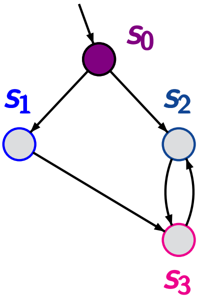

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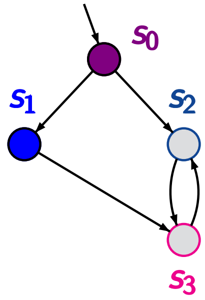

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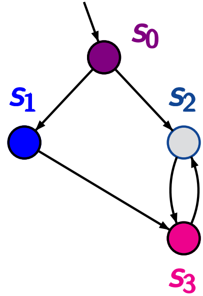

$DFS(s_0)$

$DFS(s_1)$

$DFS(s_3)$

$s_1, s_2 \not\models a$

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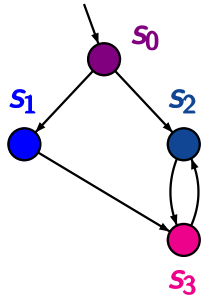

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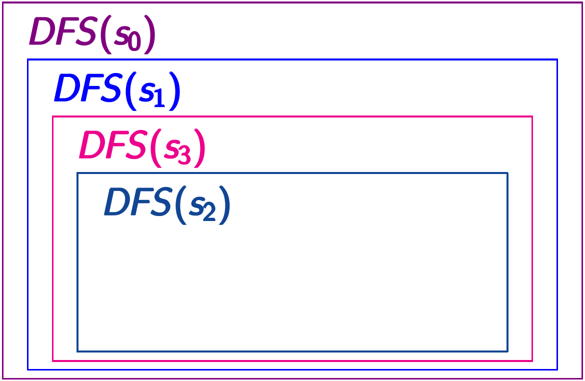

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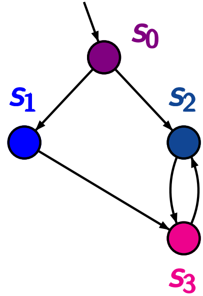

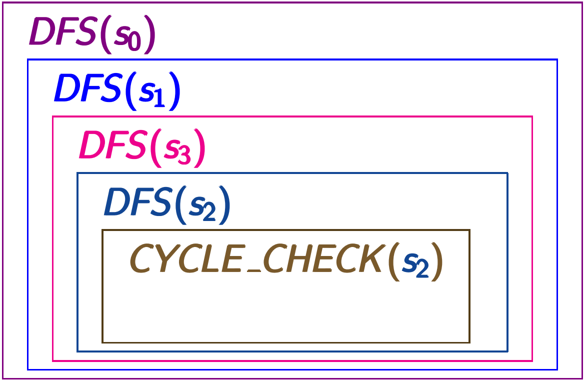

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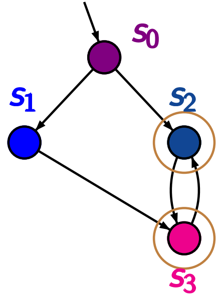

$s_1, s_2 \not\models a$

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$DFS(s_0)$

$DFS(s_1)$

$DFS(s_3)$

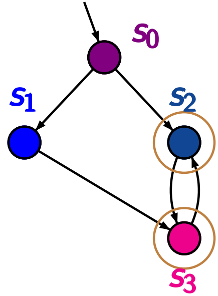
$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\rightsquigarrow$ cycle $s_2\ s_3\ s_2$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



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$\mathcal{T} \not\models$ "eventually forever a"

$DFS(s_0)$

$DFS(s_1)$

$DFS(s_3)$

$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\rightsquigarrow$ cycle $s_2\ s_3\ s_2$

↑

returns correct answer "**no**"

Nested DFS with counterexample generation

Nested DFS with counterexample generation

input: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
persistence condition $a \in AP$

output: "yes" if $\mathcal{T} \models$ "eventually forever a"
"no" + counterexample otherwise

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initial path fragment of the form

$$s_0 \ldots s_{n-1}\, s_n\, s_{n+1} \ldots s_{n+m-1}\, s_n$$

where $s_n \not\models a$

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$\uparrow$

initial path fragment of the form

$s_0 \ldots s_{n-1}\, s_n\, s_{n+1} \ldots s_{n+m-1}\, s_n$

where $s_n \not\models a$

... iterative formulation with 2 stacks ...

$U := \varnothing; \pi := \varnothing;$

$U := \varnothing;\ \pi := \varnothing; \quad \longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$U := \varnothing; \pi := \varnothing; \quad \leftarrow$ visiting set and stack for 1. DFS

$V := \varnothing; \xi := \varnothing; \quad \leftarrow$ visiting set and stack for 2. DFS

$U := \varnothing; \pi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing; \xi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

WHILE $S_0 \not\subseteq U$ DO

OD

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

WHILE $S_0 \not\subseteq U$ DO

choose $s_0 \in S_0 \setminus U$; insert s_0 in U;

OD

$U := \emptyset$; $\pi := \emptyset$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \emptyset$; $\xi := \emptyset$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

`WHILE` $S_0 \not\subseteq U$ `DO`

  choose $s_0 \in S_0 \setminus U$; insert s_0 in U; $Push(\pi, s_0)$;

`OD`

$U := \varnothing;\ \pi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing;\ \xi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊄ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);



    OD
OD
```

$U := \varnothing;\ \pi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

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WHILE S_0 ⊈ U DO
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        s := Top(π);
        IF Post(s) ⊈ U



        FI
    OD
OD
```

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    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
         THEN choose s' ∈ Post(s) \ U;
              insert s' in U; Push(π, s')



        FI
    OD
OD
```

$U := \varnothing; \pi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing; \xi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
         THEN choose s' ∈ Post(s) \ U;
              insert s' in U; Push(π, s')
         ELSE Pop(π);


        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
         THEN choose s' ∈ Post(s) \ U;
              insert s' in U; Push(π, s')
         ELSE Pop(π);
              IF s ⊭ a and CYCLE_CHECK(s)
                THEN return "no"                        FI
        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; $\leftarrow$ visiting set and stack for 1. DFS

$V := \varnothing$; $\xi := \varnothing$; $\leftarrow$ visiting set and stack for 2. DFS

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);
               IF s ⊭ a and CYCLE_CHECK(s)
                 THEN return "no" + reverse(π, ξ) FI
        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; ← visiting set and stack for 1. DFS

$V := \varnothing$; $\xi := \varnothing$; ← visiting set and stack for 2. DFS

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);
               IF s ⊭ a and CYCLE_CHECK(s)
                 THEN return "no" + reverse(π, ξ) FI
        FI
    OD
OD
return "yes"
```

Algorithm *CYCLE_CHECK*(s)

- is called for $s \not\models a$
- checks whether s belongs to a cycle
- uses global visiting set V and stack ξ

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

Algorithm $\mathit{CYCLE\_CHECK}(s)$

$\mathit{Push}(\xi, s)$; insert s in V;

$\mathtt{WHILE}\ \xi \neq \varnothing\ \mathtt{DO}$

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

    $s' := Top(\xi)$;

    IF $s \in Post(s')$

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

$s' := Top(\xi)$;

IF $s \in Post(s')$

THEN $Push(\xi, s)$; return "true"

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

  $s' := Top(\xi)$;

  IF $s \in Post(s')$

    THEN $Push(\xi, s)$; return "true"

    ELSE IF $Post(s') \not\subseteq V$

Algorithm $\mathit{CYCLE\_CHECK}(s)$

$\mathit{Push}(\xi, s)$; insert s in V;

`WHILE` $\xi \neq \varnothing$ `DO`

$\quad s' := \mathit{Top}(\xi)$;

$\quad$ `IF` $s \in \mathit{Post}(s')$

$\quad\quad$ `THEN` $\mathit{Push}(\xi, s)$; return "true"

$\quad\quad$ `ELSE` `IF` $\mathit{Post}(s') \not\subseteq V$

$\quad\quad\quad\quad$ `THEN` choose $s'' \in \mathit{Post}(s') \setminus V$;

$\quad\quad\quad\quad\quad$ insert s'' in V; $\mathit{Push}(\xi, s'')$;

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

```
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF s ∈ Post(s')
        THEN  Push(ξ, s); return "true"
        ELSE  IF Post(s') ⊈ V
                  THEN  choose s'' ∈ Post(s') \ V;
                        insert s'' in V; Push(ξ, s'');
                  ELSE  Pop(ξ)
              FI
```

```
Push(ξ, s); insert s in V;
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF s ∈ Post(s')
        THEN  Push(ξ, s); return "true"
        ELSE  IF Post(s') ⊈ V
                  THEN  choose s'' ∈ Post(s') \ V;
                        insert s'' in V; Push(ξ, s'');
                  ELSE  Pop(ξ)
              FI
    FI
OD
return "false"
```

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

```
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF  s ∈ Post(s')
        THEN    Push(ξ, s);  return "true"
        ELSE IF Post(s') ⊈ V
                THEN choose s'' ∈ Post(s') \ V;
                     insert s'' in V; Push(ξ, s'');
                ELSE Pop(ξ)
            FI
    FI
OD
return "false"
```

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

  $s' := Top(\xi)$;

  IF $\boxed{s \in Post(s')}$

    THEN $Push(\xi, s)$; return "true"

    ELSE IF $Post(s') \not\subseteq V$

      THEN choose $s'' \in Post(s') \setminus V$;

        insert s'' in V; $Push(\xi, s'')$;

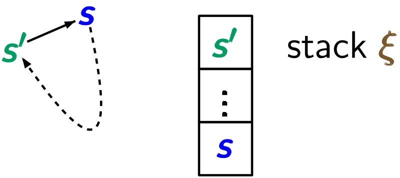
      ELSE $Pop(\xi)$

    FI

  FI

OD

return "false"



Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

  $s' := Top(\xi)$;

  IF $\boxed{s \in Post(s')}$

    THEN $\boxed{Push(\xi, s)}$ return "true"

    ELSE IF $Post(s') \not\subseteq V$

      THEN choose $s'' \in Post(s') \setminus V$;

        insert s'' in V; $Push(\xi, s'')$;

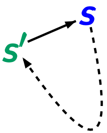
      ELSE $Pop(\xi)$

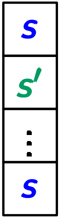
    FI

  FI

OD

return "false"





stack ξ

$U := \varnothing$; $\pi := \varnothing$; $V := \varnothing$; $\xi := \varnothing$;

WHILE $S_0 \not\subseteq U$ DO

choose $s_0 \in S_0 \setminus U$; insert s_0 in U; $Push(\pi, s_0)$;

WHILE $\pi \neq \varnothing$ DO

$s := Top(\pi)$;

IF $Post(s) \not\subseteq U$

THEN choose $s' \in Post(s) \setminus U$;

insert s' in U; $Push(\pi, s')$

ELSE $Pop(\pi)$;

IF $s \not\models a$ and $CYCLE\_CHECK(s)$

THEN return "**no**" + $reverse(\pi, \xi)$ FI

FI

OD

OD

return "**yes**"

Nested DFS with counterexample generation

$U := \varnothing$; $\pi := \varnothing$; $V := \varnothing$; $\xi := \varnothing$;

WHILE $S_0 \not\subseteq U$ DO

  choose $s_0 \in S_0 \setminus U$; insert s_0 in U; $Push(\pi, s_0)$;

  WHILE $\pi \neq \varnothing$ DO

    $s := Top(\pi)$;

    IF $Post(s) \not\subseteq U$

     THEN choose $s' \in Post(s) \setminus U$;

        insert s' in U; $Push(\pi, s')$

     ELSE $Pop(\pi)$; ⟵ $DFS(s)$ ends

        IF $s \not\models a$ and $CYCLE\_CHECK(s)$

         THEN return "**no**" + $reverse(\pi, \xi)$ FI

    FI

  OD

OD

return "**yes**"

$DFS(s)$ starts when s inserted in U

Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

V:	organizes all states that have been visited in the current and all previous calls of $CYCLE\_CHECK(\cdot)$
ξ:	stack for counterexample

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

V: organizes all states that have been visited in the current and all previous calls of $CYCLE\_CHECK(\cdot)$

ξ: stack for counterexample

soundness: 1. termination

2. partial correctness

Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

V: organizes all states that have been visited in the current and all previous calls of $CYCLE\_CHECK(\cdot)$

ξ: stack for counterexample

soundness: 1. termination $\surd$

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Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

V:	organizes all states that have been visited in the current and all previous calls of $CYCLE\_CHECK(\cdot)$
ξ:	stack for counterexample

$\mathcal{T} \models$ "eventually forever a"

iff the nested DFS returns "**yes**"

Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $\mathit{CYCLE\_CHECK}(s)$

- is called for $s \not\models a$ when $\mathit{DFS}(s)$ is finished
- uses global data structures V and ξ

V:	organizes all states that have been visited in the current and all previous calls of $\mathit{CYCLE\_CHECK}(\cdot)$
ξ:	stack for counterexample

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

Soundness of the nested DFS

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

Soundness of the nested DFS

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \rightarrow s$.

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \rightarrow s$.

Hence: s belongs to a cycle

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and $\mathit{CYCLE\_CHECK}(s)$ finds a backward edge $t \longrightarrow s$.

Hence: s belongs to a cycle and there is an ultimatively periodic path $\pi = s_0 \ldots s_{n-1}(\, s\, t_1 \ldots .\, t_k\,)^\omega$

$$\mathcal{T} \not\models \text{"eventually forever } a\text{"}$$

iff the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and $\mathit{CYCLE\_CHECK}(s)$ finds a backward edge $t \longrightarrow s$.

Hence: s belongs to a cycle and there is an ultimately periodic path $\pi = s_0 \ldots s_{n-1}(\, s\, t_1 \ldots .\, t_k\,)^\omega$ in $\mathcal{T}$ s.t. $\mathit{trace}(\pi) \notin$ "eventually forever a"

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \longrightarrow s$.

Hence: s belongs to a cycle and there is an ultimatively periodic path $\pi = s_0 \ldots s_{n-1}(\, s\, t_1 \ldots .\, t_k\,)^\omega$ in $\mathcal{T}$ s.t. $trace(\pi) \notin$ "eventually forever a"

This yields $\mathcal{T} \not\models$ "eventually forever a".

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"
iff the nested DFS returns "**no**"

Proof of "$\Longrightarrow$":

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"
iff the nested DFS returns "**no**"

Proof of "$\Longrightarrow$": show that:

When *CYCLE_CHECK*(s) is called then there is no cycle $t_0\, t_1 \ldots t_k$ in $\mathcal{T}$ s.t. $s = t_0 = t_k$ and
$t_i \in V$ for some $i \in \{1, ..., k\}$

global visiting set of the inner DFS

Soundness of the nested DFS

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

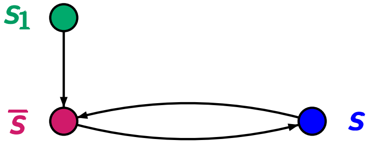
Proof of "$\Longrightarrow$": show that:

When $CYCLE\_CHECK(s)$ is called then there is no cycle $t_0\, t_1 \ldots t_k$ in $\mathcal{T}$ s.t. $s = t_0 = t_k$ and $t_i \in V$ for some $i \in \{1, ..., k\}$

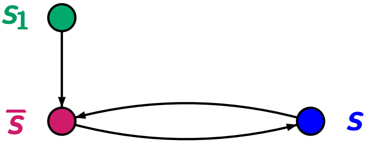
global visiting set of the inner DFS

Hence: if s belongs to a cycle then $CYCLE\_CHECK(s)$ will find a backward edge $t \rightarrow s$

Correctness of the nested DFS



Correctness of the nested DFS



$DFS(s_1)$

$\boxed{DFS(s) \quad CC(s)}$

$CC(s_1)$

$DFS(s)$ $CC(s)$

$DFS(s_1)$ $CC(s_1)$

$DFS(s)$

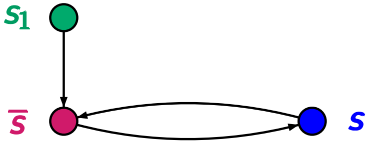
$\boxed{DFS(s_1) \quad CC(s_1)}$

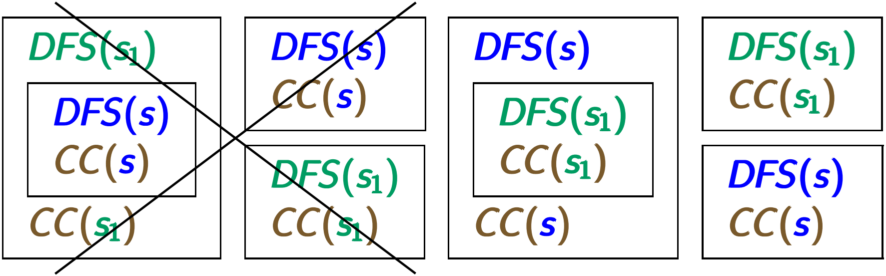
$CC(s)$

$DFS(s_1)$ $CC(s_1)$

$DFS(s)$ $CC(s)$

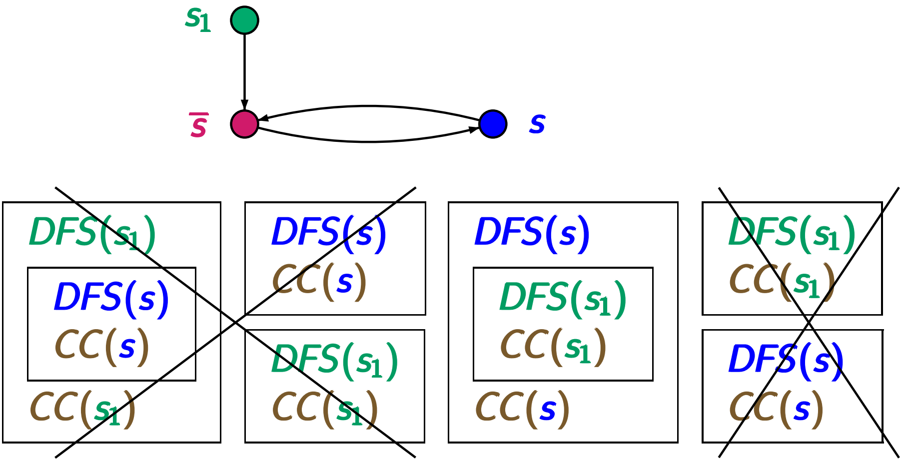
Correctness of the nested DFS





as $CC(s_1)$ is called
before $CC(s)$

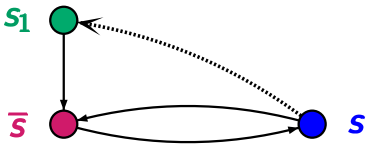
Correctness of the nested DFS



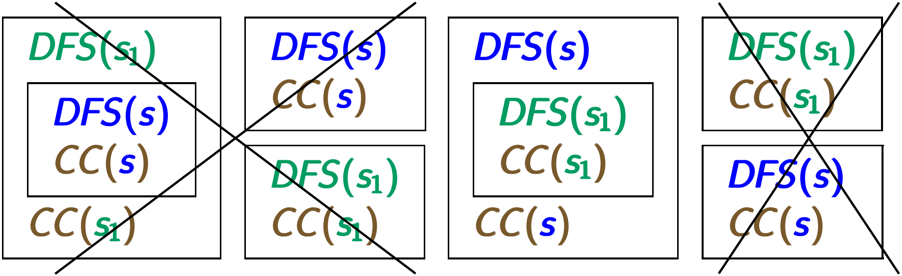
as $CC(s_1)$ is called before $CC(s)$

as s is reachable from s_1

Correctness of the nested DFS



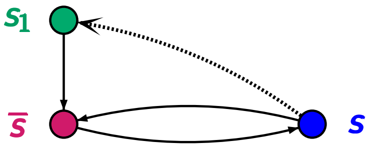
s_1 reachable from s



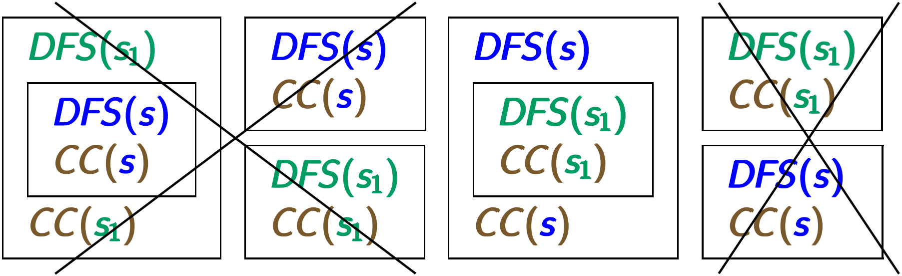
as $CC(s_1)$ is called before $CC(s)$

as s is reachable from s_1

Correctness of the nested DFS



s_1 reachable from s



as $CC(s_1)$ is called before $CC(s)$

$CC(s_1)$ would have found the cycle $s_1 \rightsquigarrow \overline{s} \rightsquigarrow s \rightsquigarrow s_1$

as s is reachable from s_1

Further improvements of the nested DFS

$U := \varnothing;\ \pi := \varnothing;$

$V := \varnothing;\ \xi := \varnothing;$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U;
        ⋮
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
            THEN ...
            ELSE Pop(π);
             IF s ⊭ a and CYCLE_CHECK(s) THEN ...
        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
    choose $s_0 \in S_0 \setminus U$;
        $\vdots$
    WHILE $\pi \neq \varnothing$ DO
        $s := Top(\pi)$;
        IF $Post(s) \not\subseteq U$
            THEN ...
            ELSE $Pop(\pi)$;
             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
        FI
    OD
OD

on the fly construction

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
    choose $s_0 \in S_0 \setminus U$;
        $\vdots$
    WHILE $\pi \neq \varnothing$ DO
        $s := Top(\pi)$;
        IF $Post(s) \not\subseteq U$
            THEN ...
            ELSE $Pop(\pi)$;
             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
        FI
    OD
OD

on the fly construction

hash techniques for U and V

Further improvements of the nested DFS

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
    choose $s_0 \in S_0 \setminus U$;
    $\vdots$
    WHILE $\pi \neq \varnothing$ DO
        $s := Top(\pi)$;
        IF $Post(s) \not\subseteq U$
            THEN ...
            ELSE $Pop(\pi)$;
             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
        FI
    OD
OD

on the fly construction

hash techniques for U and V

$$\underbrace{s \in U \setminus V}_{\langle s, 1\rangle} \qquad \underbrace{s \in V}_{\langle s, 0\rangle}$$

Further improvements of the nested DFS

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
    choose $s_0 \in S_0 \setminus U$;
    $\vdots$
    WHILE $\pi \neq \varnothing$ DO
        $s := Top(\pi)$;
        IF $Post(s) \not\subseteq U$
            THEN ...
            ELSE $Pop(\pi)$;
             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
        FI
    OD
OD

on the fly construction

hash techniques for U and V

$$\underbrace{s \in U \setminus V}_{\langle s, 1 \rangle} \qquad \underbrace{s \in V}_{\langle s, 0 \rangle}$$

early termination of $CYCLE\_CHECK$, e.g., if a state in π is visited