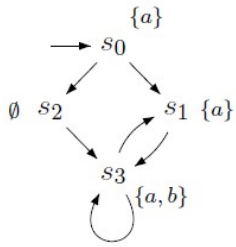
MVS – Esercizi proposti - 28 marzo 2013

Esercizio 1

Consider the transition system given below. Formally define its traces!



Esercizio 2

Consider the set AP of atomic propositions defined by $AP = \{x = 0, x > 1\}$ and consider a non-terminating sequential computer program P that manipulates the variable x over the domain $\mathbb{N}$. Formulate the following informally stated properties as LT properties:

- false and true
- initially x is equal to zero
- initially x differs from zero
- x exceeds one only finitely many times
- the value of x alternates between zero and one
- initially x is equal to zero, but at some point x exceeds one

Determine which of these LT properties are safety properties.

Esercizio 3

Let $AP = \{a, b\}$ and let P be the LT property of all infinite words $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ such that there exists $n \geq 0$ with $a \in A_i$ for $0 \leq i < n$, $\{a, b\} = A_n$ and $b \in A_j$ for infinitely many $j \geq 0$. Provide a decomposition $P = P_{safe} \cap P_{live}$ into a safety and into a liveness property.

Esercizio 4

Let P denote the set of traces of the form $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ such that

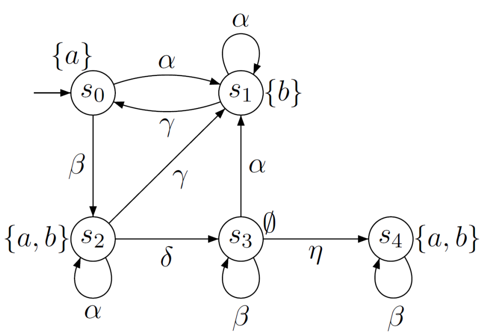
$$\overset{\infty}{\exists} k.\ A_k = \{a, b\} \quad \wedge \quad \exists n \geq 0.\ \forall k > n.\ \left(a \in A_k \Rightarrow b \in A_{k+1}\right).$$

Consider the following fairness assumptions with respect to the transition system TS outlined on the right:

a) $\mathcal{F}_1 = \left(\{\{\alpha\}\}, \{\{\beta\}, \{\delta, \gamma\}, \{\eta\}\}, \emptyset\right)$.
Decide whether $TS \models_{\mathcal{F}_1} P$.

b) $\mathcal{F}_2 = \left(\{\{\alpha\}\}, \{\{\beta\}, \{\gamma\}\}, \{\{\eta\}\}\right)$.
Decide whether $TS \models_{\mathcal{F}_2} P$.

Justify your answers!



Esercizio 5

Let $AP = \{a, b, c\}$. Consider the following linear time properties:

(a) If a becomes valid, afterwards b stays valid ad infinitum or until c holds.

(b) Between two neighbouring occurrences of a, b always holds.

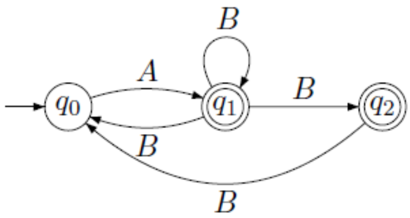
(c) Between two neighbouring occurrences of a, b occurs more often than c.

(d) $a \wedge \neg b$ and $b \wedge \neg a$ are valid in alternation or until c becomes valid.

For each property P_i $(1 \leq i \leq 4)$, decide if it is a regular safety property (argument why!) and if so, define the NFA $\mathcal{A}_i$ with $\mathcal{L}(\mathcal{A}_i) = BadPref(P_i)$.
Hint: You may use propositional formulas over the set AP as transition labels.

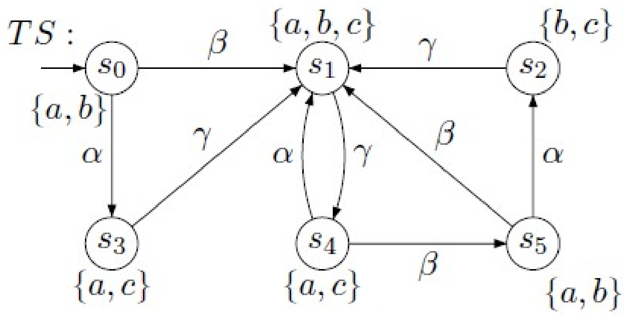
Esercizio 6

Consider the GNBA outlined on the right with acceptance sets $F_1 = \{q_1\}$ and $F_2 = \{q_2\}$. Construct an equivalent NBA using the transformation introduced in the lecture.



Esercizio 7

Consider the following transition system TS



and the regular safety property

$$P_{safe} = \text{“always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before, then } a \text{ and } b \text{ do not hold thereafter at least until } c \text{ holds”}$$

As an example, it holds:

$$\{b\}\emptyset\{a,b\}\{a,b,c\} \in pref(P_{safe})$$
$$\{a,b\}\{a,b\}\emptyset\{b,c\} \in pref(P_{safe})$$
$$\{b\}\{a,c\}\{a\}\{a,b,c\} \in BadPref(P_{safe})$$
$$\{b\}\{a,c\}\{a,c\}\{a\} \in BadPref(P_{safe})$$

Questions:

(a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = MinBadPref(P_{safe})$.

(b) Decide whether $TS \models P_{safe}$ using the $TS \otimes \mathcal{A}$ construction.
Provide a counterexample if $TS \not\models P_{safe}$.