# Models of computation (MOD) 2016/17
Exam – February 13, 2018

**[Ex. 1]** Let IMP$^-$ be the fragment of IMP without assignments.

1. Prove by structural induction that in IMP$^-$ $\forall c.\ P(c)$, where

$$P(c) \stackrel{\text{def}}{=} \forall \sigma, \sigma'.\ \langle c, \sigma\rangle \to \sigma' \ \Rightarrow\ \sigma' = \sigma$$

2. Let $div : \mathbf{Com} \to \mathbf{Bexp}$ defined by

$$div(\mathbf{skip}) \stackrel{\text{def}}{=} \mathbf{false} \qquad div(c_1; c_2) \stackrel{\text{def}}{=} div(c_1) \vee div(c_2) \qquad div(\mathbf{while}\ b\ \mathbf{do}\ c) \stackrel{\text{def}}{=} b$$

$$div(\mathbf{if}\ b\ \mathbf{then}\ c_1\ \mathbf{else}\ c_2) \stackrel{\text{def}}{=} (b \wedge div(c_1)) \vee (\neg b \wedge div(c_2))$$

Prove by rule induction that in IMP$^-$ $\forall c, \sigma, \sigma'.\ Q(\langle c, \sigma\rangle \to \sigma')$, where

$$Q(\langle c, \sigma\rangle \to \sigma') \stackrel{\text{def}}{=} \mathcal{B}\,[\![div(c)]\!]\,\sigma = \mathbf{false}$$

*Hint:* exploit point 1 above in the proof

**[Ex. 2]** Let $\mathbb{N}^\infty \stackrel{\text{def}}{=} \mathbb{N} \cup \{\infty\}$ and let $\wp_f(X)$ be the set of <u>finite</u> subsets of $X$.

1. Show that the PO$_\perp$ $(\wp_f(\mathbb{N}^\infty), \subseteq)$ is not complete.
2. Prove that the PO$_\perp$ $(\wp_f(\mathbb{N}^\infty) \cup \{\mathbb{N}^\infty\}, \subseteq)$ is complete.
3. Note that $(\mathbb{N}^\infty, \geq)$ defines a CPO$_\perp$, where $\geq$ is the usual "greater than or equal to" relation extended by letting $\infty \geq n$ for any $n \in \mathbb{N}^\infty$. Prove that the function $min : (\wp_f(\mathbb{N}^\infty) \cup \{\mathbb{N}^\infty\}, \subseteq) \to (\mathbb{N}^\infty, \geq)$ that returns the smallest element in a set (with $min(\emptyset) = \infty$) is monotone but not continuous.

**[Ex. 3]** Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\alpha.x\ +\ \beta.\mathbf{nil}) \qquad q \stackrel{\text{def}}{=} \mathbf{rec}\ y.(\overline{\alpha}.y\ +\ \overline{\beta}.\mathbf{nil})$$

$$r \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\tau.z\ +\ \tau.\mathbf{nil}) \qquad s \stackrel{\text{def}}{=} \mathbf{rec}\ w.(\tau.w\ +\ \tau.\tau.\mathbf{nil})$$

1. Draw the LTS of the process $t \stackrel{\text{def}}{=} (p|q)\backslash\alpha\backslash\beta$.
2. Prove that $t$ and $r$ are strong bisimilar.
3. Show that $t$ and $s$ are not strong bisimilar.

**[Ex. 4]** A certain experiment is believed to be described by a two-state DTMC with the transition matrix $P$ below, where the parameter $0 < p < 1$ is unknown:

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ p & 1-p \end{pmatrix}$$

When the experiment is run many times it is observed that it ends in state one 20 percent of the time and in state two 80 percent of the time. Compute a sensible estimate of the parameter $p$.