ES 1

Monday, March 2, 2020 10:54 AM

LINDA PAGLI uff. 277

Notazione asintotica

O $O(f(n))$

Ω $\Omega(f(n))$

Θ limite esatto

IS caso ottimo $\Theta(n)$ } $O(n^2)$

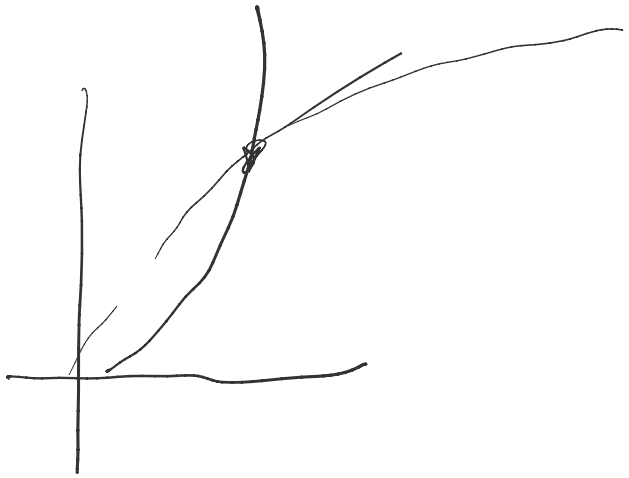
caso pessimo $\Theta(n^2)$ }

SS $\Theta(n^2)$

$f(n) = 3n^2$

$O(n^2)$?

$O(n^4)$? sì



$O(n \log n)$? no

$O(n^2 \log n)$?

$O(n)$? no

$\Omega(n)$? sí

$\Omega(\sqrt{n})$? sí

$\Omega(n^2)$? sí

$\Omega(n^2 \log n)$ no

$\Omega(n^3)$? no

$\Theta(n^2)$? sí

$\Theta(n)$? no

$\Theta(n \log n)$? no

$\Theta(n^2 \log n)$? no

MERGE SORT è $O(n \log n)$
$\Omega(n \log n)$

$\Theta(n \log n)$

$\Theta(n^k)$

↓

complessità polinomiale

k costante

$\Theta(k^{g(n)})$ ||

$\Theta(n^{g(n)})$

$\Theta(1)$ costante

↓

esponenziale !

?

$P \neq NP$

$\$ \; 1'000'000$

$\log^2 n$, 3^{n-2}, π^n, $n^5 - 5n^2$, $n^4 - 7n^3$,

$n^{\log n}$, $\frac{n}{\log n}$, $\sqrt{n}$, 19, $(\log n)^n$

19

$\log^2 n$

$\sqrt{n} = \frac{n}{\sqrt{n}} < \frac{n}{\log n}$

$\frac{n}{\log n}$

$n^4 - 7n^3$

$n^5 - 5n^2$

2) $3^{n-2} = \left(2^{\log_2 3}\right)^{n-2} = 2^{(\log_2 3)(n-2)}$

4) $\pi^n = 2^{\log_2 \pi \cdot n}$

1) $n^{\log n} = \left(2^{\log_2 n}\right)^{\log n} = 2^{\log^2 n}$

4) $(\log n)^n = \left(2^{\log \log n}\right)^n = 2^{n \log \log n}$

SEARCH and SORT

a n K

input: a di n interi, K

output: posizione di k di a
oppure $k \notin a$ (-1 pos)

Ricerca sequenziale:

caso pessimo: $\Theta(n)$

caso ottimo: $\Theta(1)$

caso medio:

si calcola la media

k è in prima pos. 1 confronto
k è in sec pos. 2 confronti

⋮ ⋮

$a[i] = k$?

k è in posiz. n n

k è in posiz. : n $\quad$ n

k non $\notin a$ $\quad$ n

$$C = \underbrace{1 + 2 + \dots + n}_{n+1} + n =$$

$$\sum_{i=1}^{n} i = \frac{(n+1)\,n}{2}$$

formula di Gauss

1 2 3 . . . . . 99 99 100

101

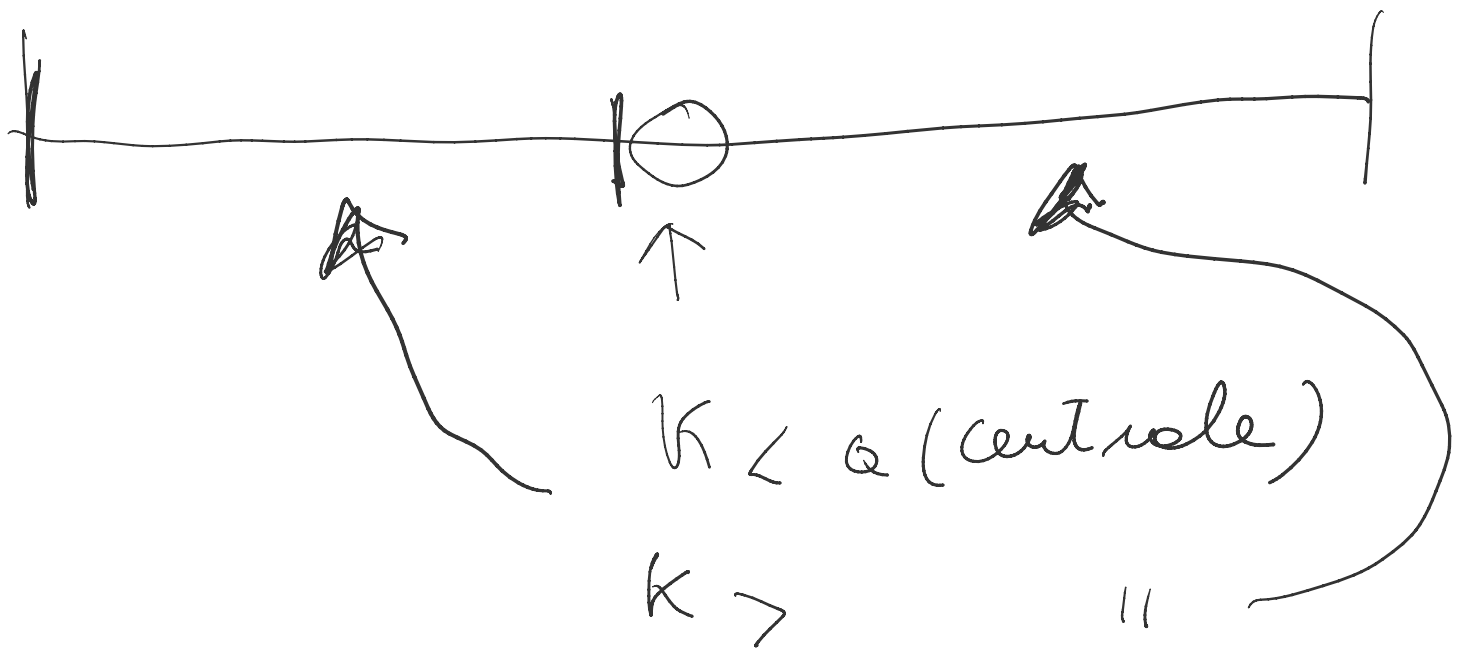
$$= \frac{n(n+1)}{2(n+1)} + \frac{\overset{\sim 1}{n}}{n+1} \cong \frac{n}{2} + 1 = \Theta(n)$$

Ricerca Binaria

input = a di n el. ordinato in ordine crescente (non decrescente)

K.

preprocessing $\Theta(n \log n)$



RicercaBinaria1 (a, k):

sin = 1 ; des = n;

while (sin ≤ des) {

cen = $\frac{sin + des}{2}$;

if (k == a[cen]) return cen

else (if k < a[cen]) des = cen - 1;

else sin = cen + 1 ;

} $\Theta(1)$

return -1;

caso pessimo:

n	1° ciclo
n/2	2° ciclo
n/4	⋮
⋮	⋮
$n/2^i = 1$	i-esimo ciclo

$$\frac{n}{2^i} = 1 \qquad n = 2^i$$

$$\log n = \log_2 2^i$$

$$\boxed{i = \log n}$$

Ricerca Binaria

$O(\log n)$

Divide et Impera per

- Dividi
- Risolvi $\begin{cases} \text{ricorsivamente} \\ \text{direttamente} \end{cases}$
- ~~Combina~~ ?

1° elemento Ricerca Binaria $2(a, 1, 1, n)$

- ~~compito~~ 1° dicembre RicercaBinaria2(a, 1, 1, n)

RicercaBinaria2(a, K, sx, dx):

if sx > dx return -1;

cx = (sx + dx) / 2;

if(K == a[cx]) return cx;

if(K < a[cx]) return RicBin2(a, K, sin, cx-1);

else return RicBin2(a, K, cx+1, dx);

$$T(n) = \begin{cases} \Theta(1) & \text{se } n = 0 \\ T\left(\frac{n}{2}\right) + \Theta(1) \end{cases}$$

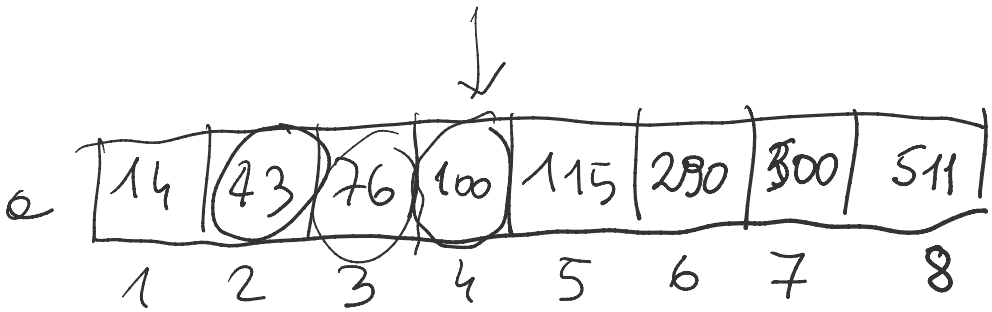
$a = 1 \quad b = 2 \quad f(n) = n^0$

$n^{\log_2 1} : n^0 \Rightarrow$

$n^{\alpha_2} : n^{\alpha_1}$

$n^0 : n^0 \Rightarrow$ caso 2

$$T(n) = \Theta(\log n)$$



$K = 76$

gx	dx	cx
1	8	4
1	3	2
3	3	3

$K < 100$

$K > 43$

$K = 76$ $\quad cx = 3$