

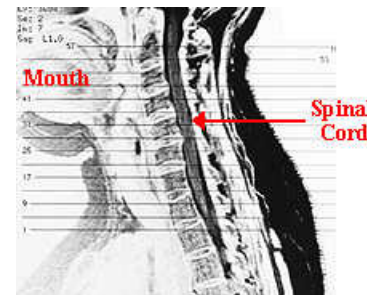
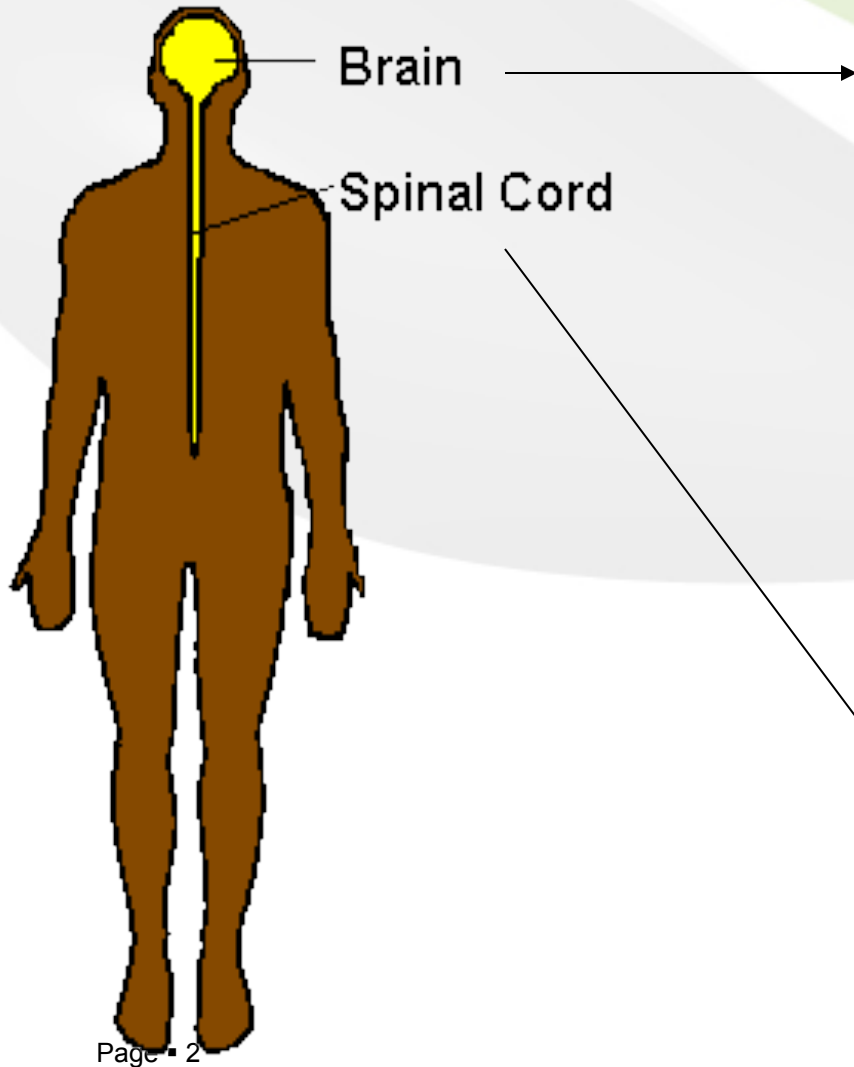


University of Pisa

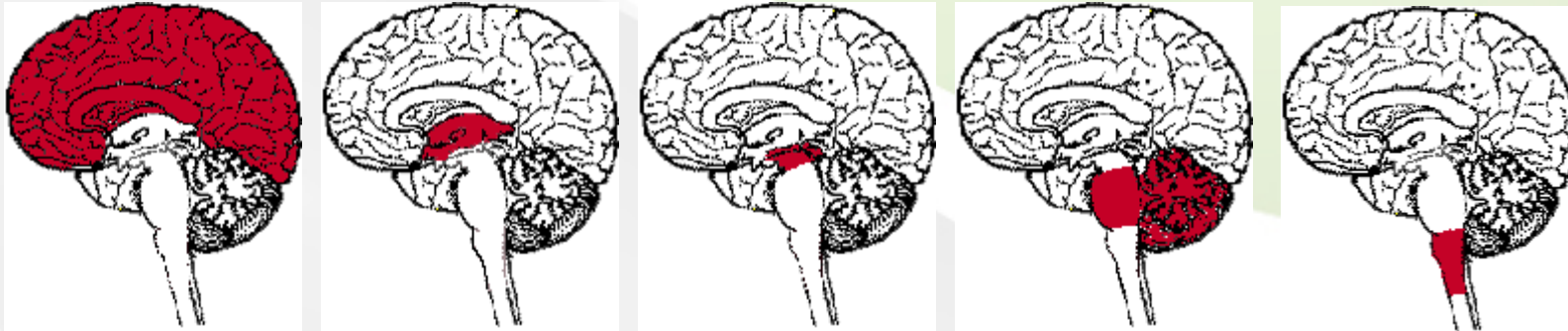
# Neural and Neuron-Astrocyte Modeling

Gaetano Valenza

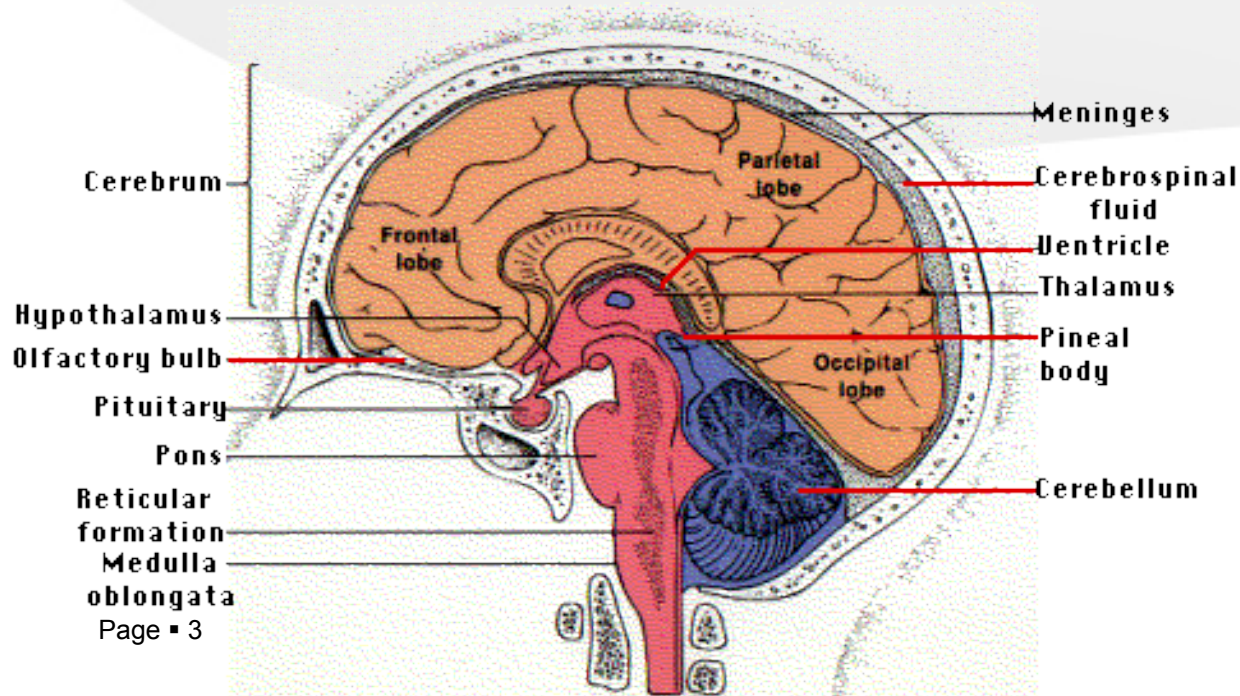
# Central Nervous System (CNS)



# Encephalon: Anatomical division

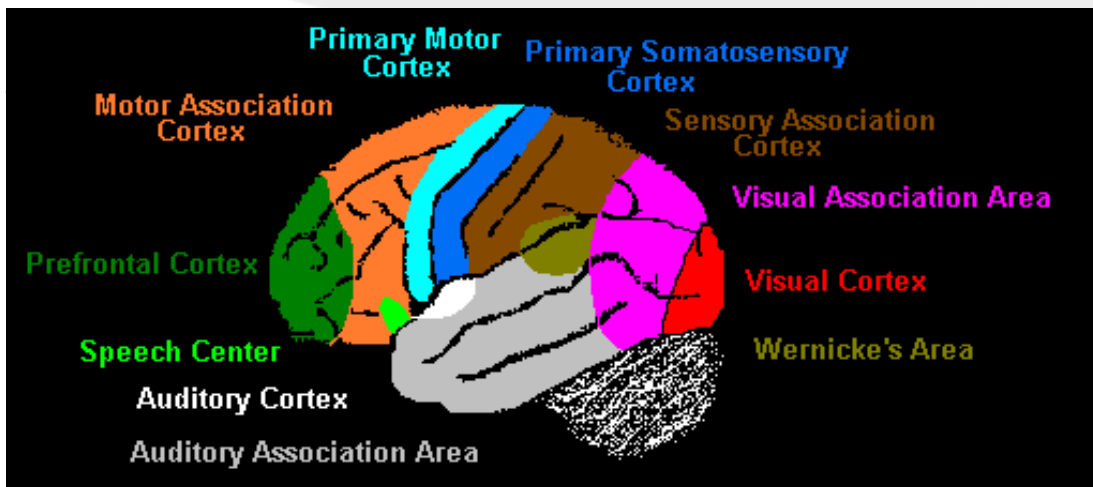
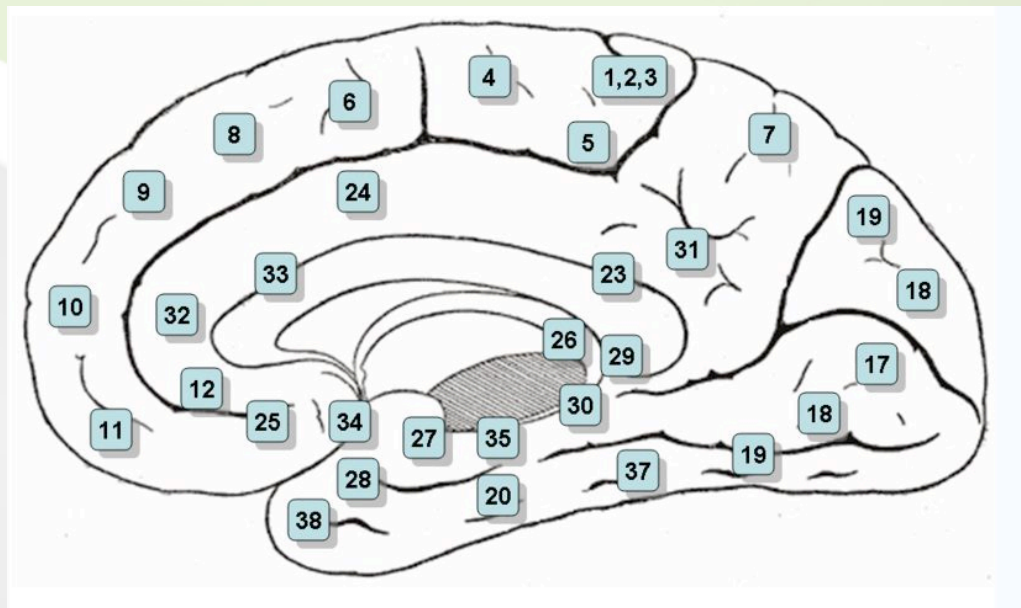


Telencephalon    Diencephalon    Mesencephalon    Metencephalon    Myelencephalon

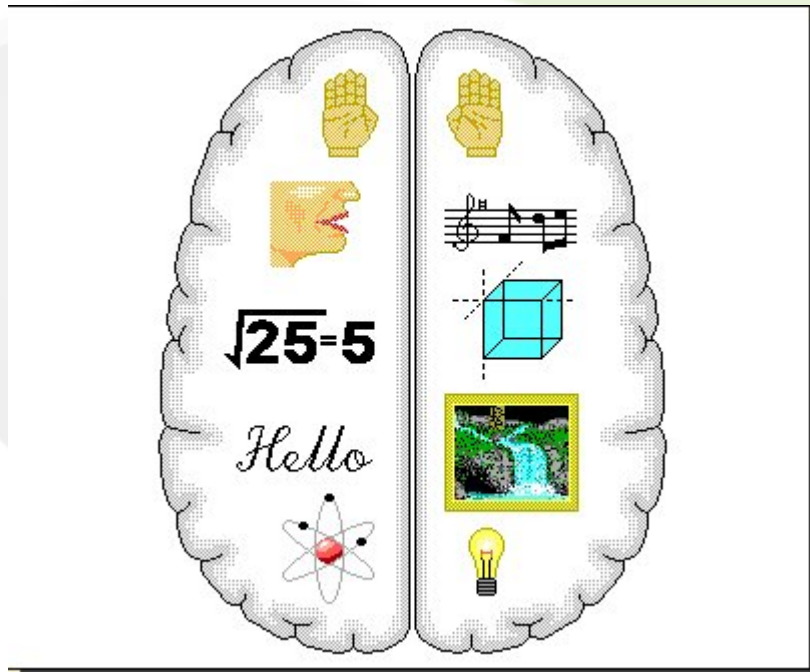


# The Cerebral Cortex: Functional Division

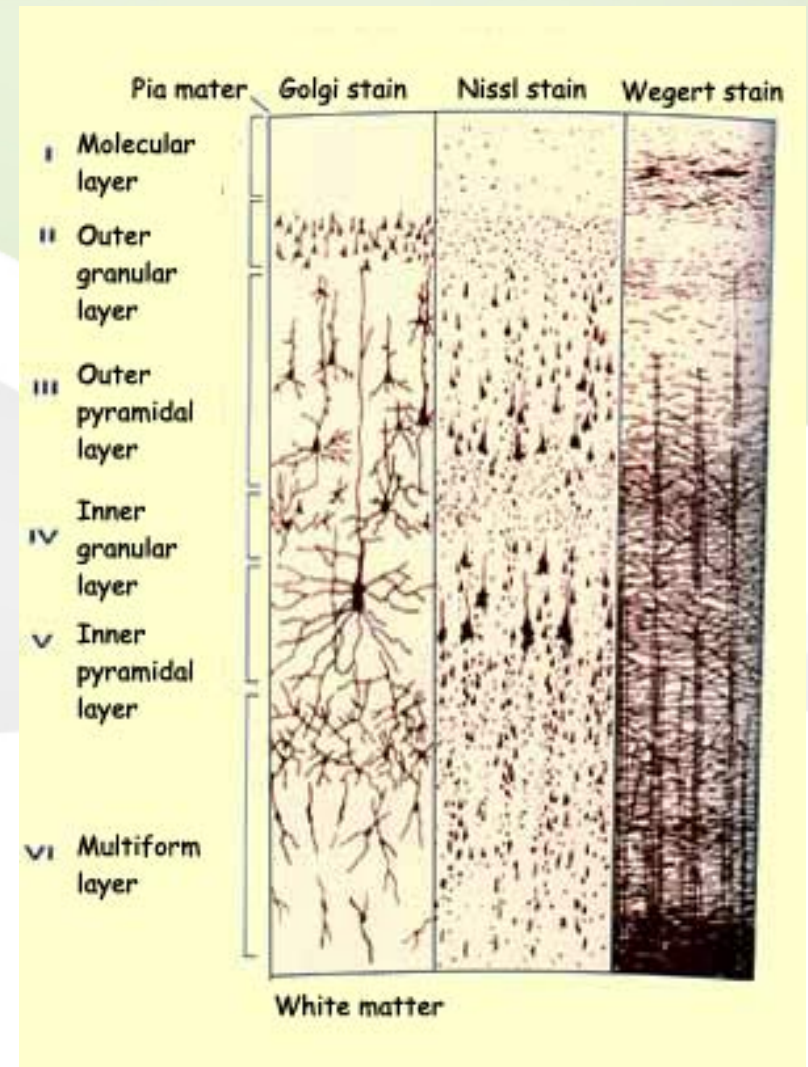
## Brodmann Areas



# The Cerebral Cortex (1)

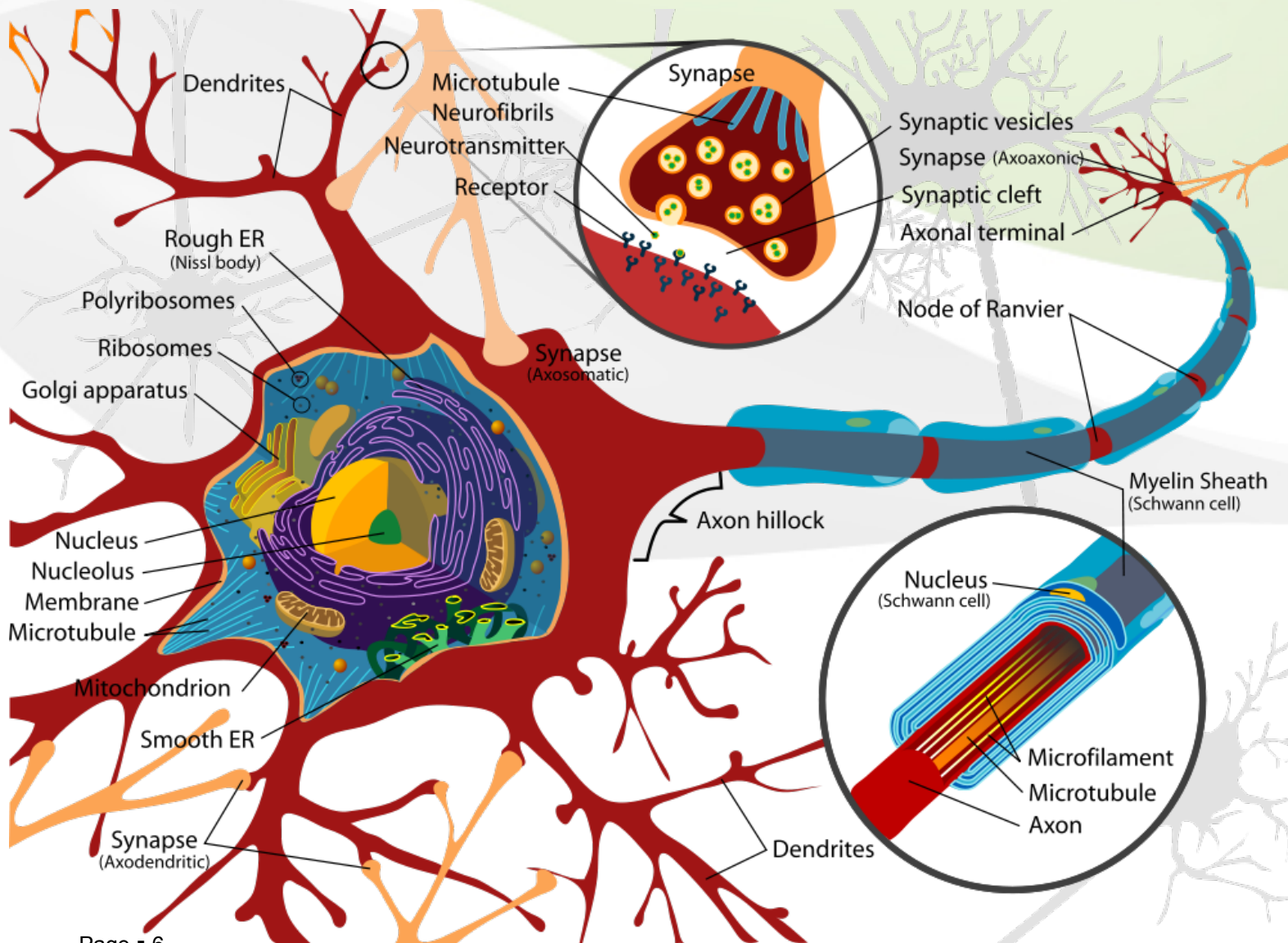


The cerebral cortex is responsible of many cognitive functions such as language, memory, emotional processing, etc.



Six layers of neurons

# Modelling Neural Dynamics



# Levels of Modeling

Brain as a whole

Specific brain systems (visual system,...)

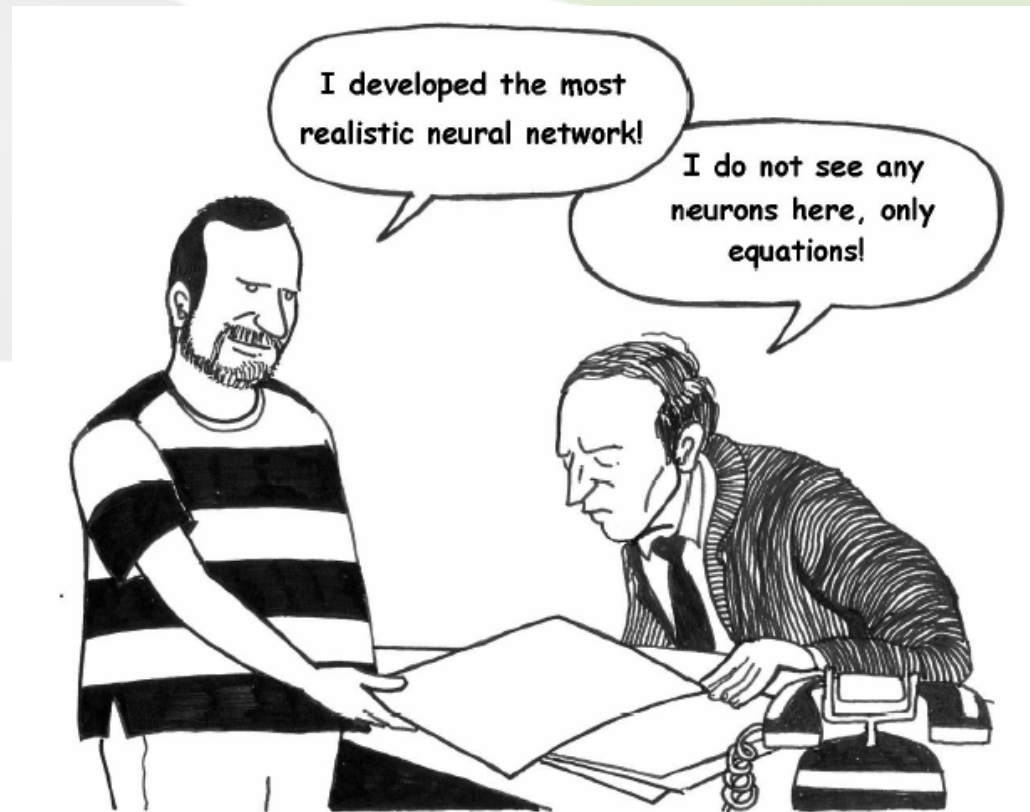
Large scale neural networks

Small neural networks

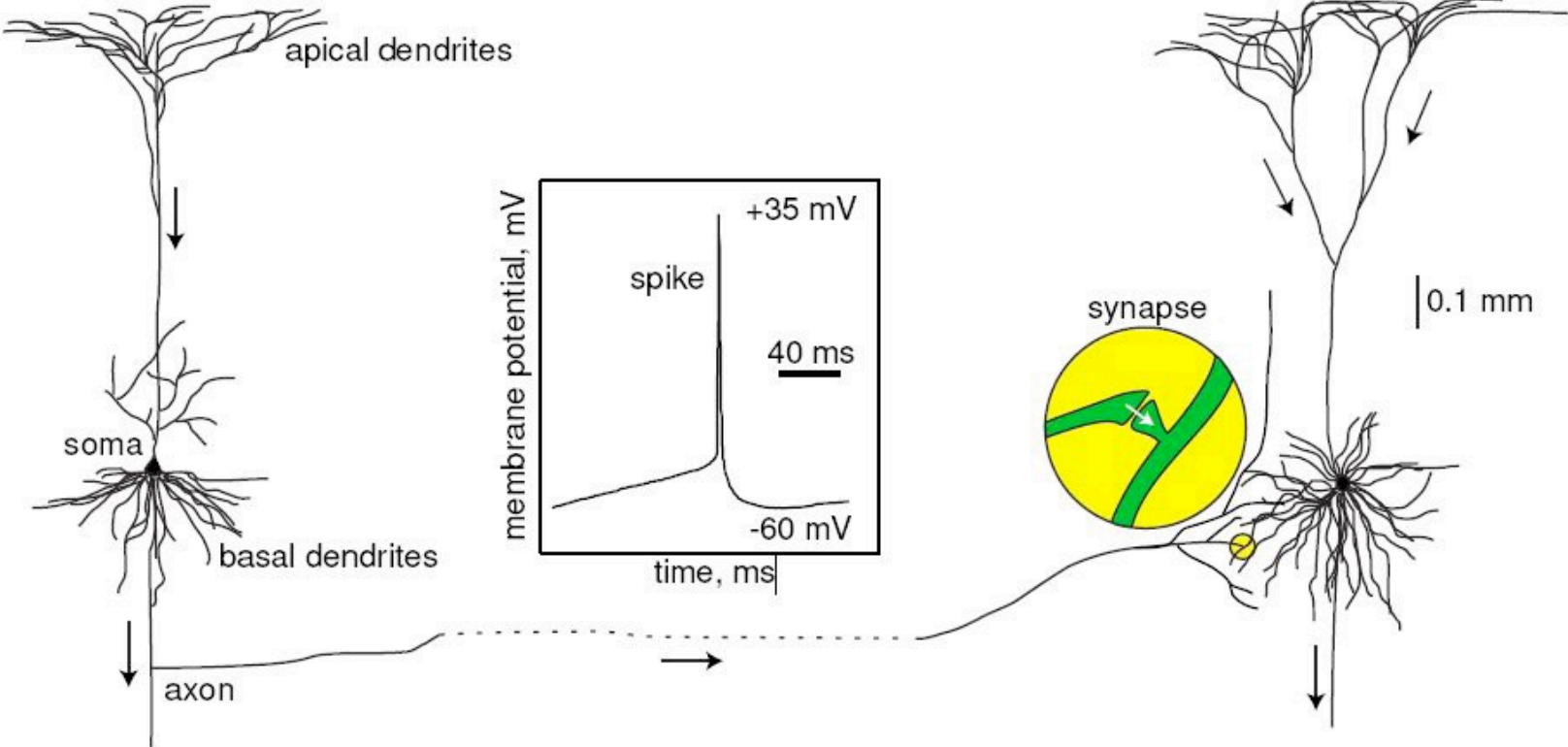
Neurons

Ion channels and synapses

Molecular processes

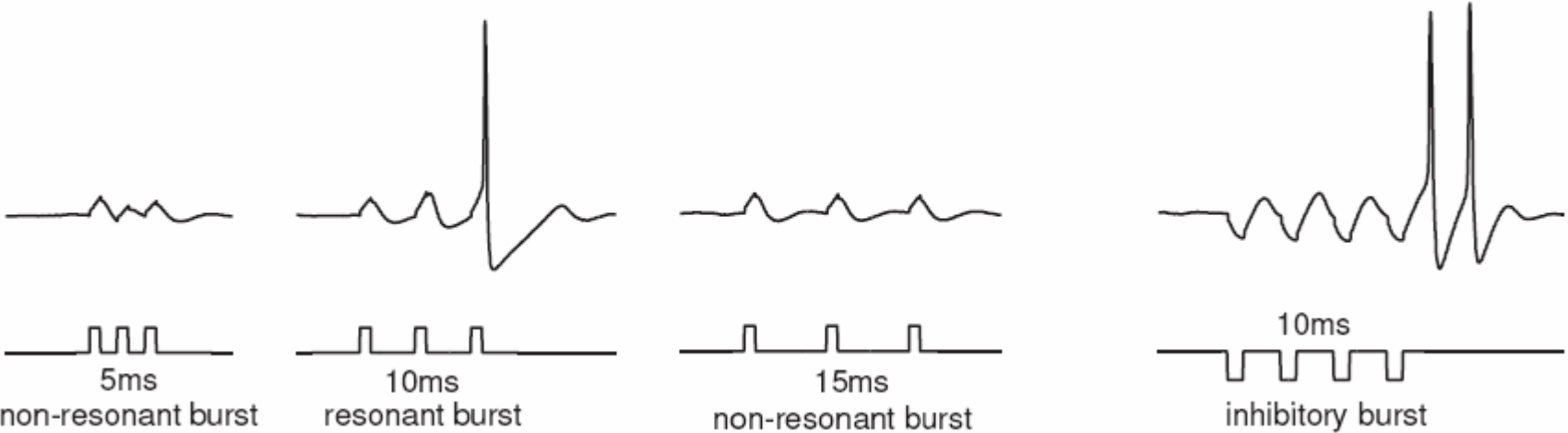
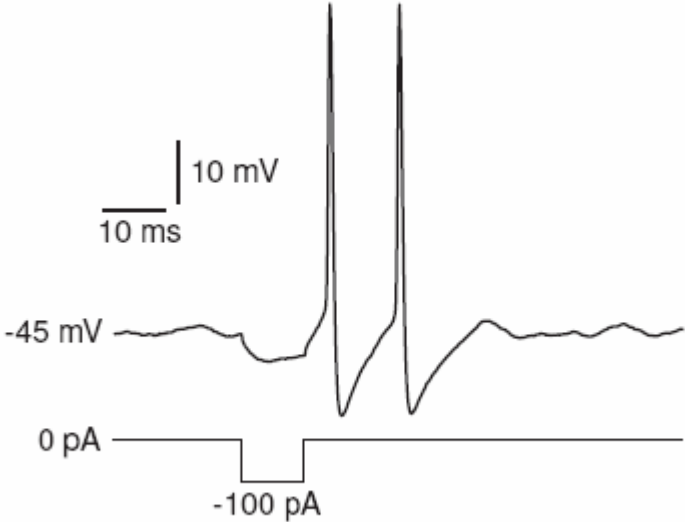


# Neural Spiking

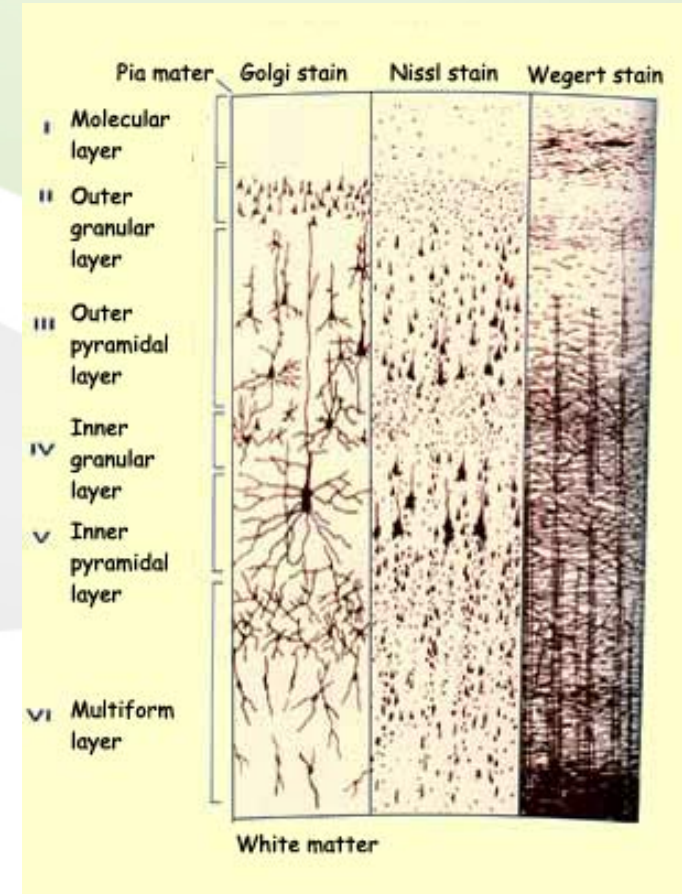
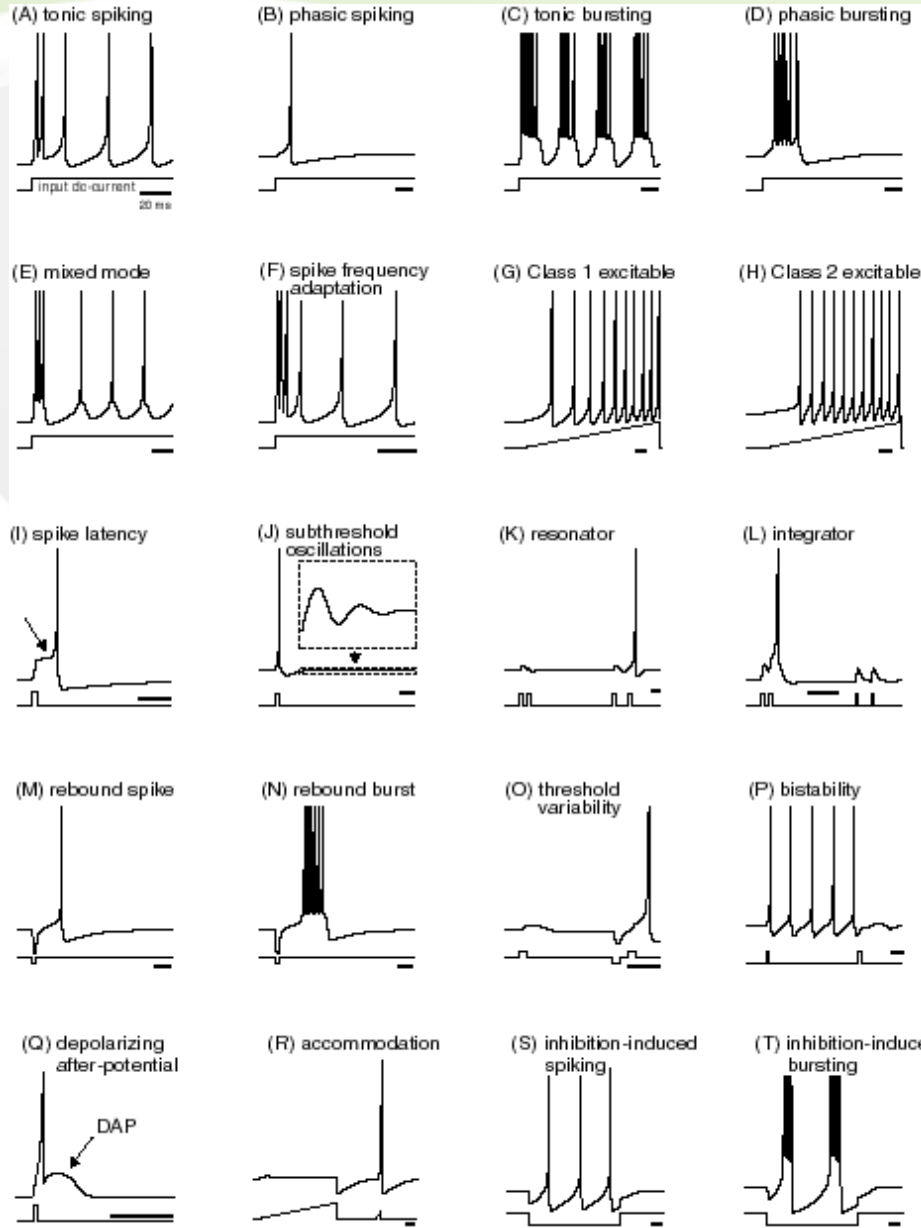




# Neural Spiking

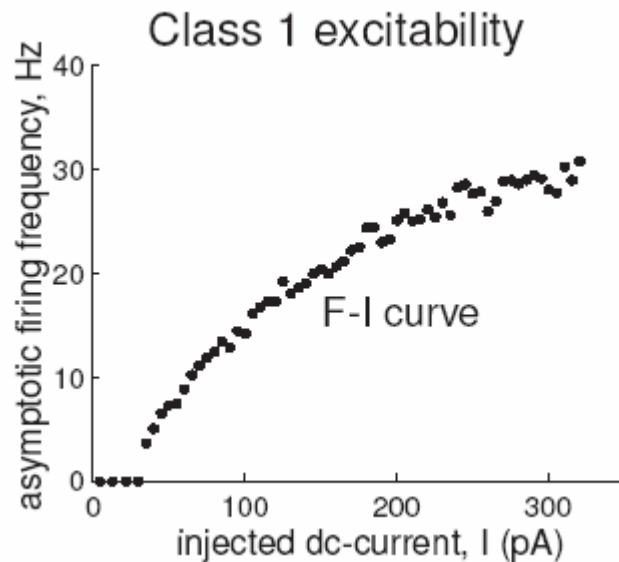


# Particular Neural Dynamics

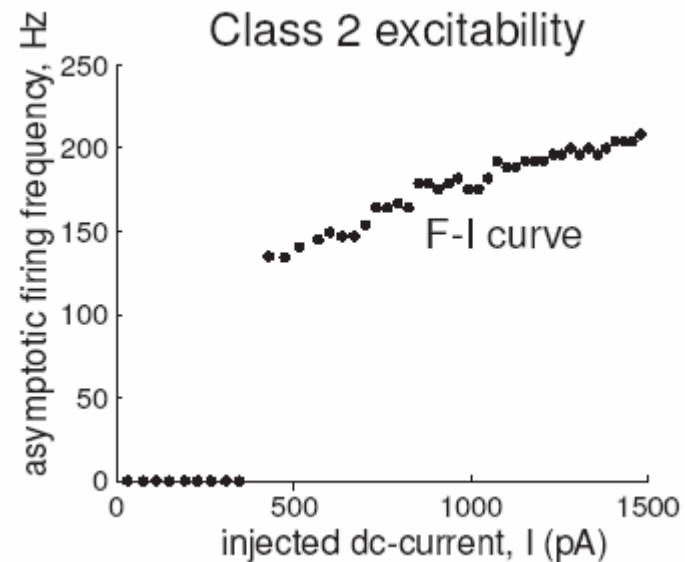


# Hodgking classification of neural excitability

- CLASS 1 NEURAL EXCITABILITY. Action potentials can be generated with arbitrarily low frequency, depending on the strength of the applied current.
- CLASS 2 NEURAL EXCITABILITY. Action potentials are generated in a certain frequency band that is relatively insensitive to changes in the strength of the applied current.
- CLASS 3 NEURAL EXCITABILITY. A single action potential is generated in response to a pulse of current. Repetitive (tonic) spiking can be generated only for extremely strong injected currents or not at all.



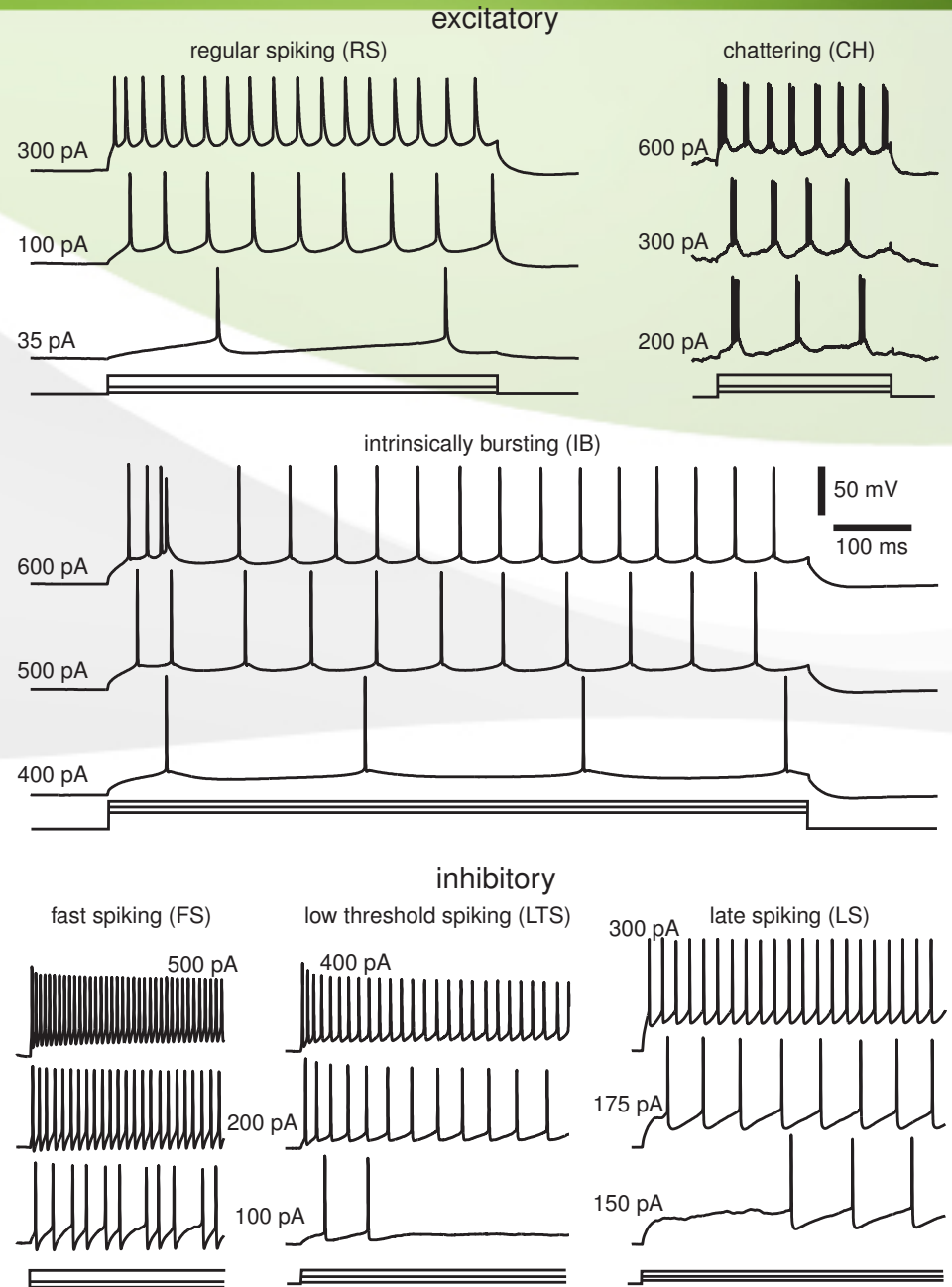
e.g. in cortical pyramidal neurons



e.g. brainstem mesV

# Particular Neural Dynamics in the Neocortex

**Six most fundamental classes** of firing patterns of neocortical neurons in response to pulses of depolarizing dc-current. RS and IB are in vitro recordings of pyramidal neurons of layer 5 of primary visual cortex of a rat, CH was recorded in vivo in cat's visual cortex. FS was recorded in vitro in rat's primary visual cortex, LTS was recorded in vitro in layer 4 or 6 of rat's barrel cortex. LS was recorded in layer 1 of rat's visual cortex.



## Benefits

- Can reproduce activity of single neurons
- Can be used to model detailed changes (external currents or the effect of drugs)

## Disadvantages

- Needs neuron morphology (dendritic layout)
- Needs information about ion channels, synapse position, neurotransmitter type
- Is slow to calculate for large numbers of neurons

=> Need for simplified neuron models

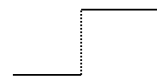
# Modelling Neural Dynamics

## The McCulloch-Pitts neuron (1943)

$$h = \sum_i x_i^{\text{in}}$$

Summation of input (no synaptic weights!)

$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$



Step-wise activation function

-> Birth of artificial neural network (ANN) research

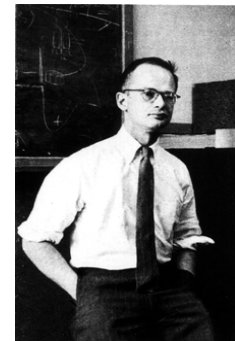
BULLETIN OF  
MATHEMATICAL BIOPHYSICS  
VOLUME 5, 1943

### A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH AND WALTER PITTS

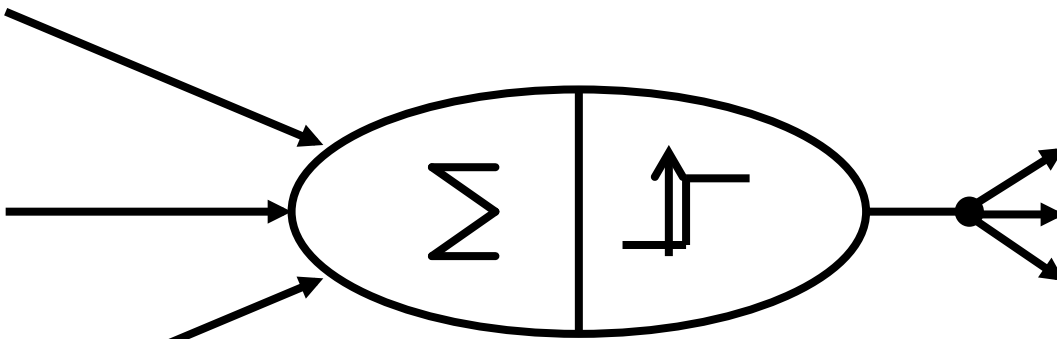
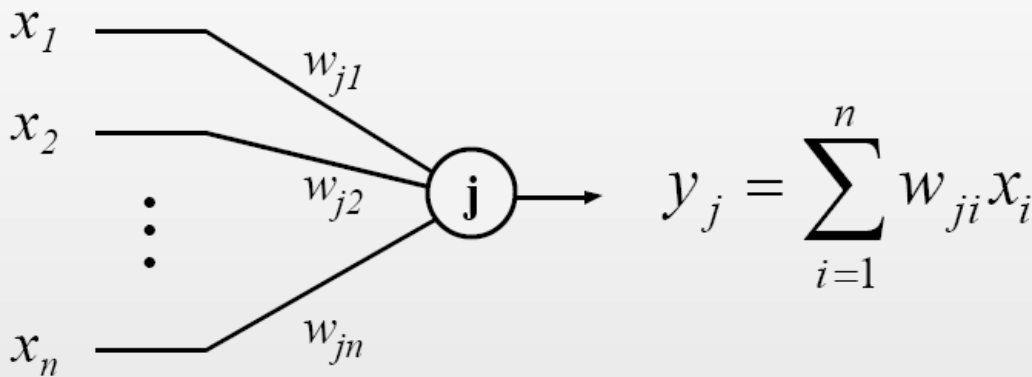
FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,  
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INS  
AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, r  
events and the relations among them can be treated by means of p  
sitional logic. It is found that the behavior of every net can be des  
in these terms, with the addition of more complicated logical mean  
nets containing circles; and that for any logical expression satis



# Modelling Neural Dynamics

## The first artificial neuron

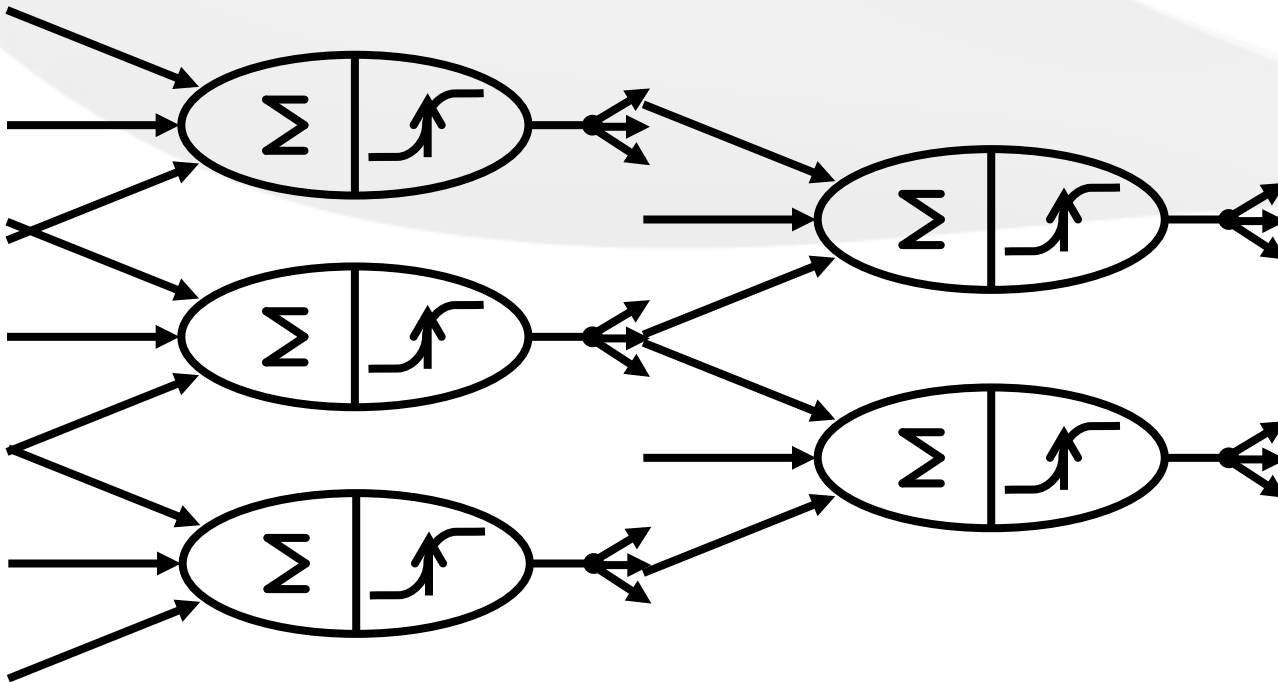


$$y_j = \begin{cases} 1, & \text{if } \sum_{i=1}^n w_{ji} x_i \geq \theta \\ 0, & \text{if } \sum_{i=1}^n w_{ji} x_i < \theta \end{cases}$$

# Modelling Neural Dynamics

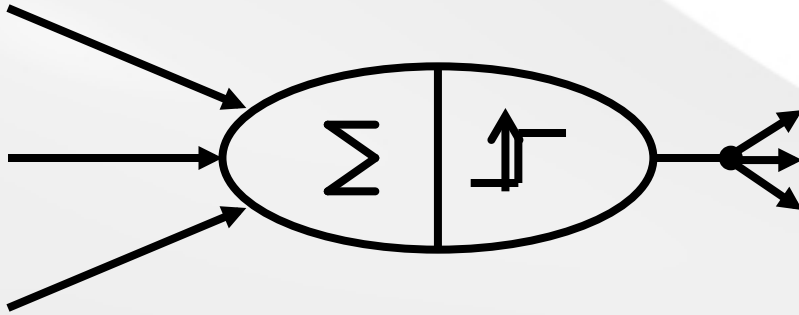
$$y_j = f\left(\sum_{i=1}^n w_{ji} x_i\right)$$

Multilayered  
Perception is a  
**universal  
approximator**



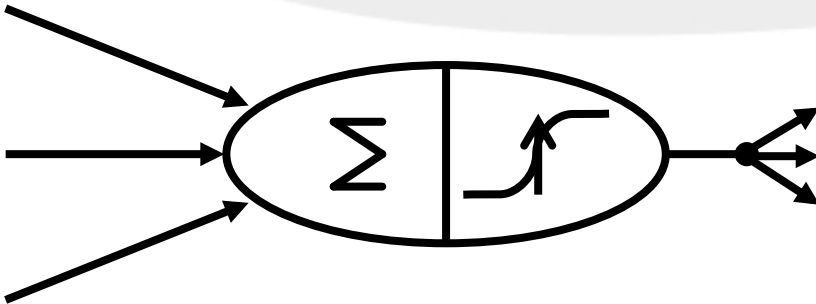


# Modelling Neural Dynamics



$$y = \begin{cases} 1 & \text{spike occurrence} \\ 0 & \text{spike absence} \end{cases}$$

From neurophysiology point of view,  $y$  is **existence of an output spike**



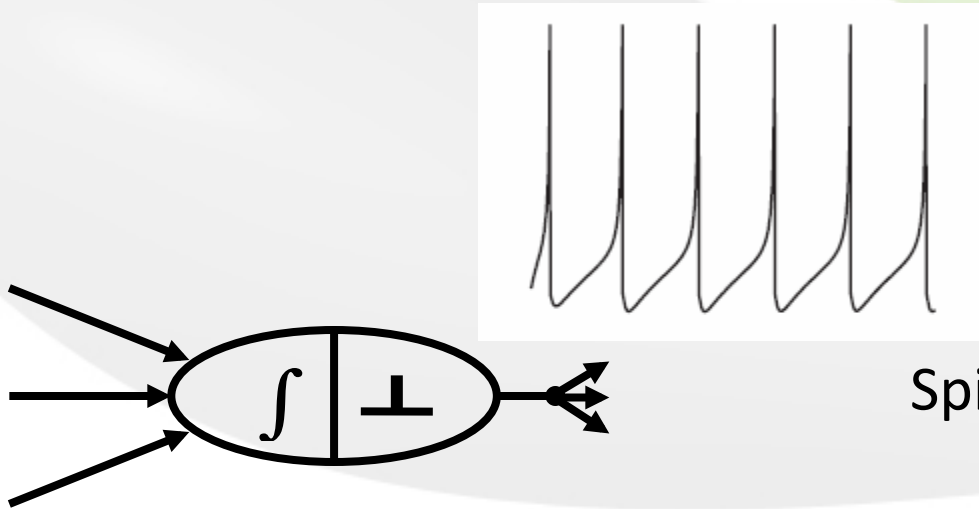
$$y = \frac{\text{Number of spikes}}{\text{Time frame}}$$

From neurophysiology point of view,  $y$  is **firing rate**

Spike timing is **not considered** at all!

# Modelling Neural Dynamics

## Spiking neuron model



Spiking neural networks are

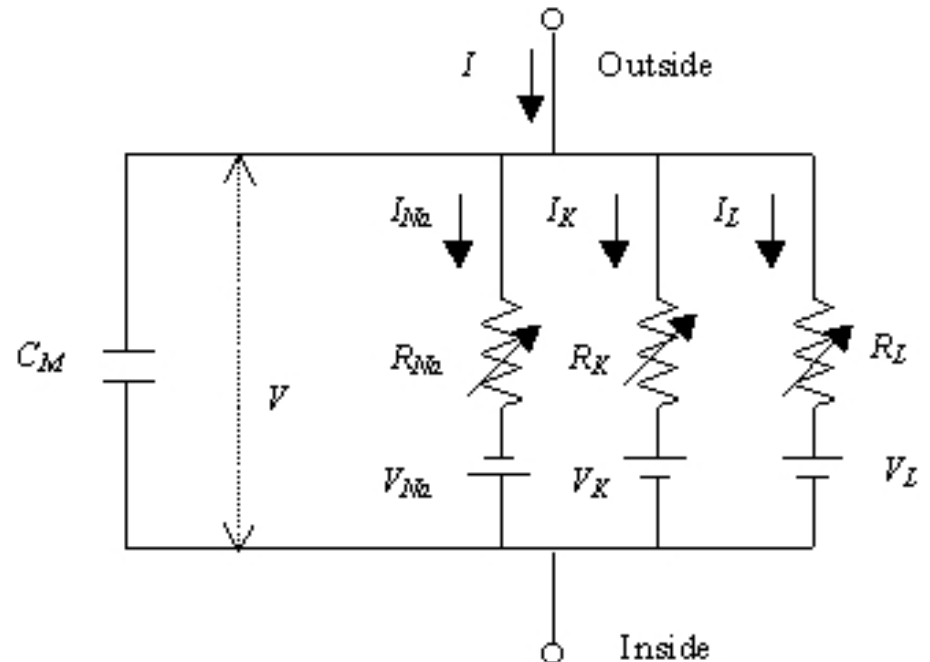
- biologically **more plausible**,
- computationally **more powerful**,
- considerably **faster**

than networks of the second generation

# Modelling Neural Dynamics

## Hodgkin-Huxley (first biologically-plausible neural model - 1952)

$$C \dot{V} = I - \overbrace{\bar{g}_K n^4 (V - E_K)}^{I_K} - \overbrace{\bar{g}_{Na} m^3 h (V - E_{Na})}^{I_{Na}} - \overbrace{g_L (V - E_L)}^{I_L}$$
$$\dot{n} = \alpha_n(V)(1 - n) - \beta_n(V)n$$
$$\dot{m} = \alpha_m(V)(1 - m) - \beta_m(V)m$$
$$\dot{h} = \alpha_h(V)(1 - h) - \beta_h(V)h ,$$



# Hodgkin-Huxley model

$$\frac{dv}{dt} = \frac{I_{external} - (I_K + I_{Na} + I_{leak})}{C}$$

$$I_K = g_K n^4 (v - V_K)$$

$$I_{Na} = g_{Na} m^3 h (v - V_{Na})$$

$$I_{leak} = g_{leak} (v - V_{leak})$$

$$\frac{dm}{dt} = \alpha_m(v)(1 - m) - \beta_m(v)m$$

$$\frac{dn}{dt} = \alpha_n(v)(1 - n) - \beta_n(v)n$$

$$\frac{dh}{dt} = \alpha_h(v)(1 - h) - \beta_h(v)h$$

$$\alpha_m(v) = 0.1(v + 25) / (e^{(v+25)/10} - 1)$$

$$\alpha_n(v) = 0.01(v + 10) / (e^{(v+10)/10} - 1)$$

$$\alpha_h(v) = 0.07e^{v/20}$$

$$\beta_m(v) = 4e^{v/18}$$

$$\beta_n(v) = 0.125e^{v/80}$$

$$\beta_h(v) = 1 / (e^{(v+30)/10} + 1)$$

Sign is wrong  
in the paper  
from 1952!

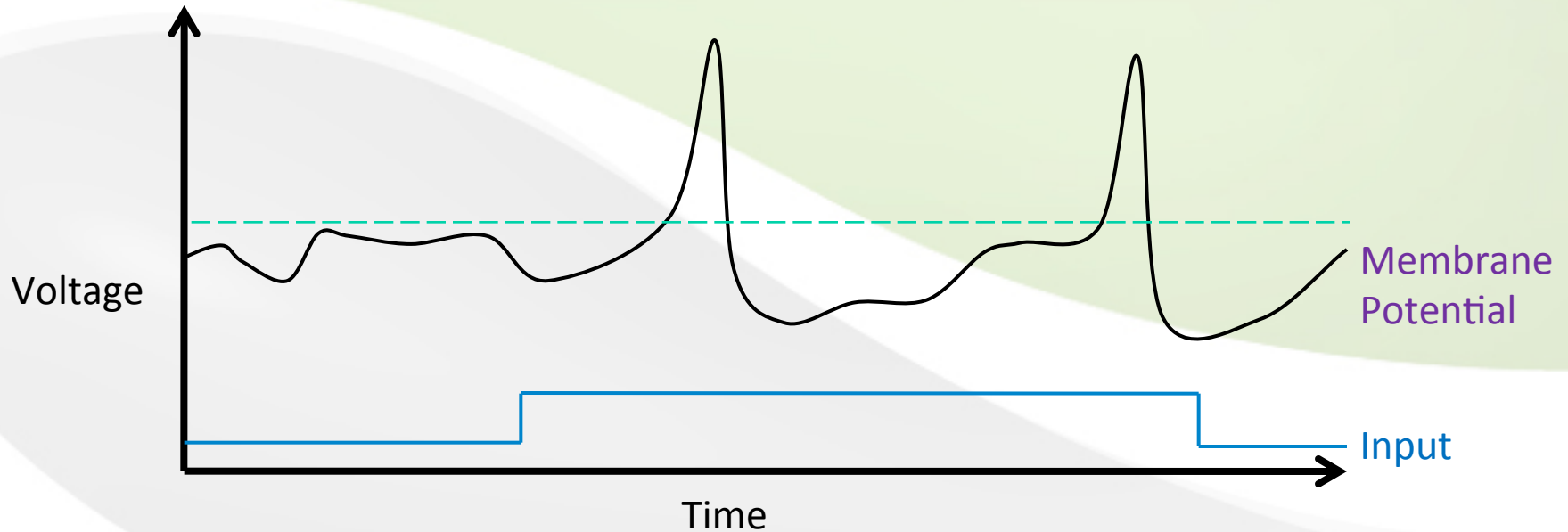
K conductance:	$g_K$	= 36
Na conductance:	$g_{Na}$	= 120
Leak conductance:	$g_{leak}$	= 0.3
Membrane Capacitance:	$C$	= 1
K equilibrium:	$V_K$	= 12
Na equilibrium:	$V_{Na}$	= -115
Leak equilibrium:	$V_{leak}$	= -10.6

Initial and Rest potential  $v_0 = 0$

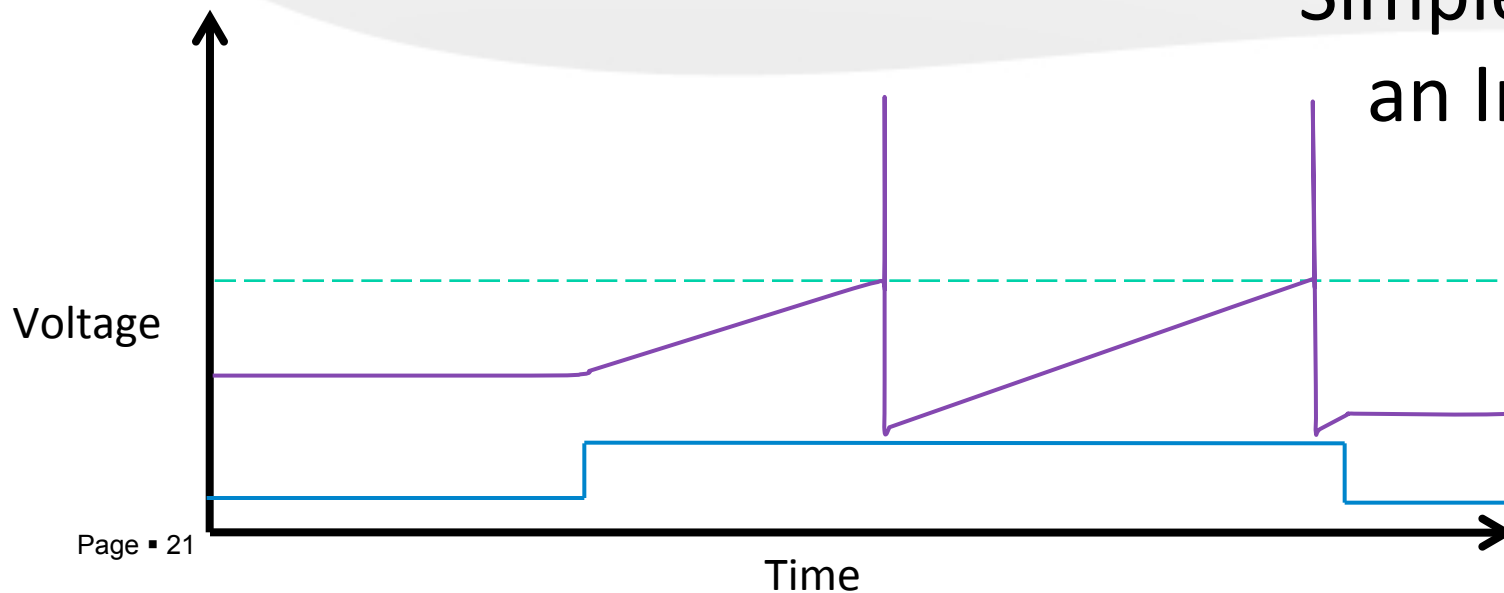
Initial channel activations  $m_0, n_0, h_0 = 0$



# What does a neuron do?



Simplest idea –  
an Integrator



# A neuron as an Integrator

Input current:

$I$

Membrane Capacitance:

$C$

Spike threshold:

$V_{thresh}$

Reset voltage:

$V_{reset}$

Membrane Voltage:

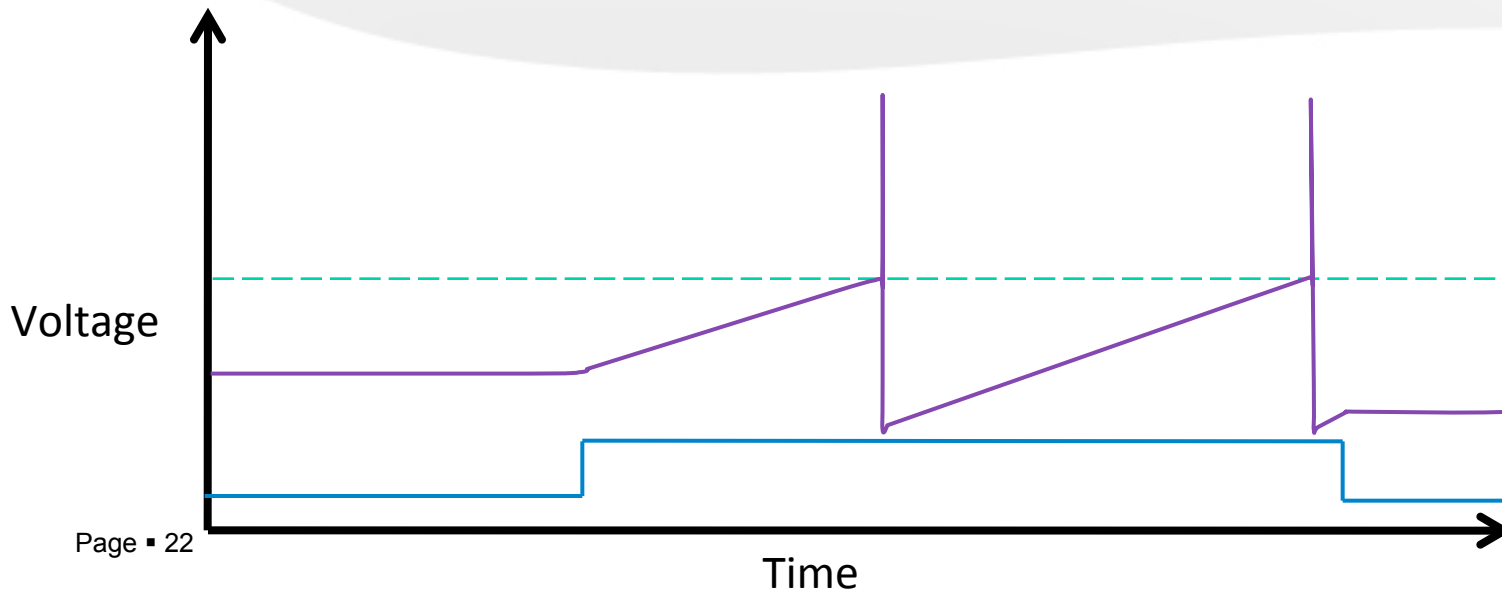
$$\frac{dv}{dt} = \frac{I}{C}$$

if  $v > V_{thresh}$   
 $\rightarrow v = V_{reset}$

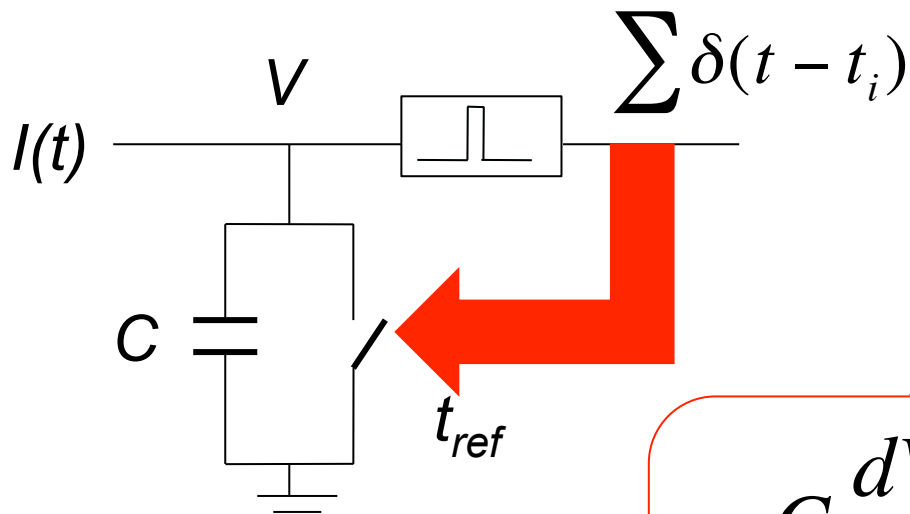
- Firing rate is unlimited
- Integration is “perfect”



Neuron response is linear



# A neuron as an Integrator



$$C \frac{dV(t)}{dt} = I(t)$$

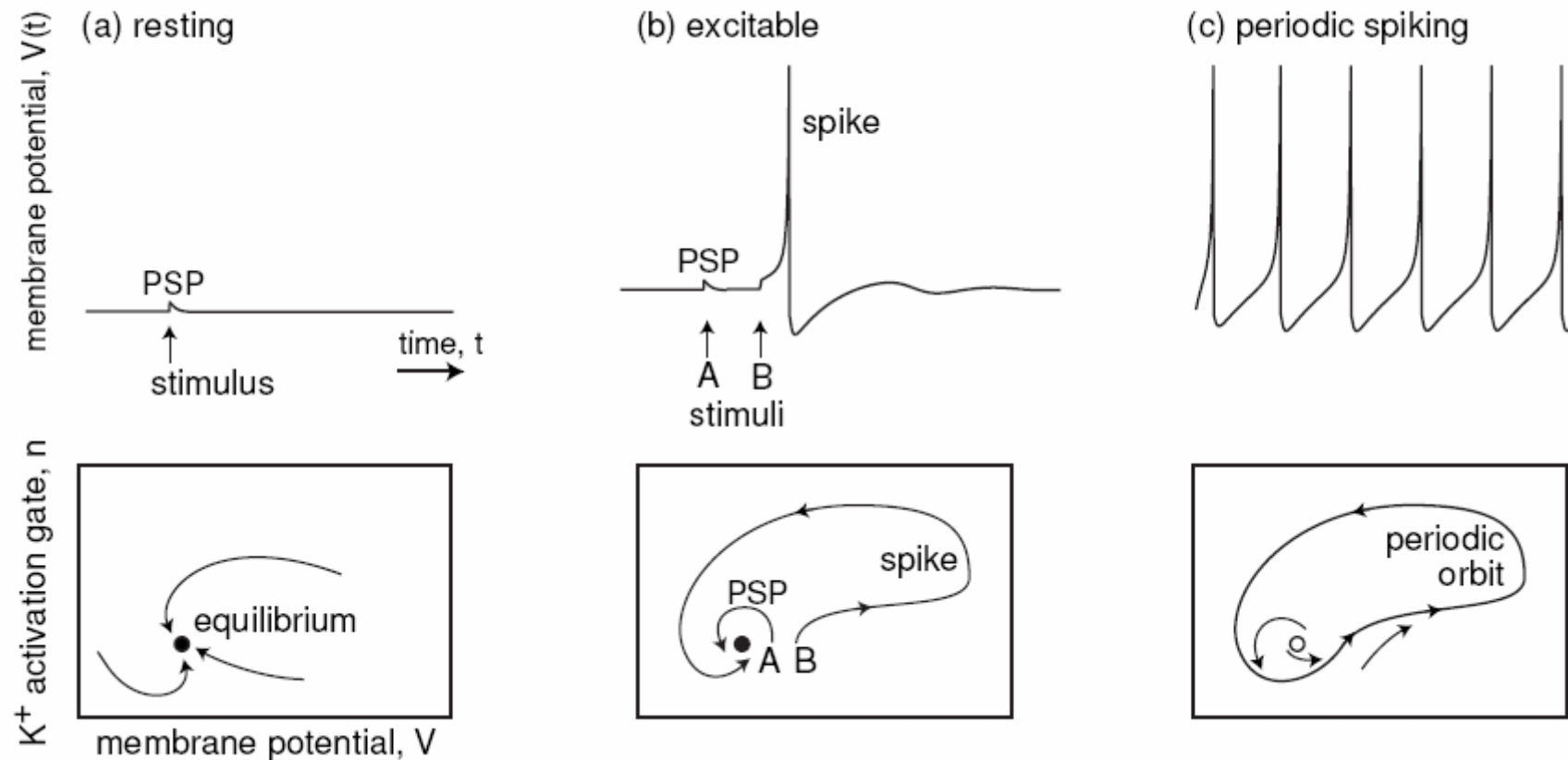
*linear*

$$V(t) = V_{Thr} \Rightarrow \text{Fire+reset } \textit{threshold}$$

$$\int_{t_i}^{t_{i+1}} I(t) dt = CV_{th}$$

# Neural Modeling and Dynamics

## Neurons as dynamical systems: phase space



- Neurons are dynamical systems.
- Resting state of neurons corresponds to a stable equilibrium, tonic spiking state corresponds to a limit cycle attractor.
- Neurons are excitable because the equilibrium is near a bifurcation.



# Neural Excitability

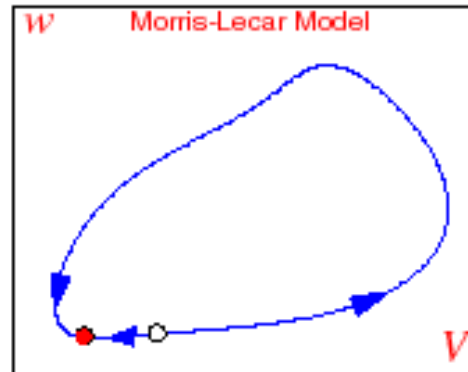
Excitability is the **most fundamental** property of **neurons** allowing **communication via action potentials or spikes**.

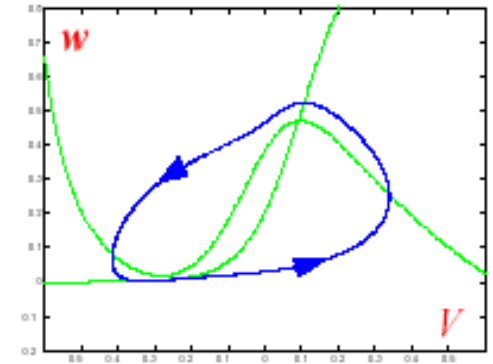
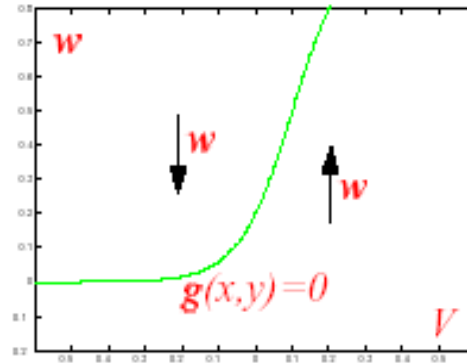
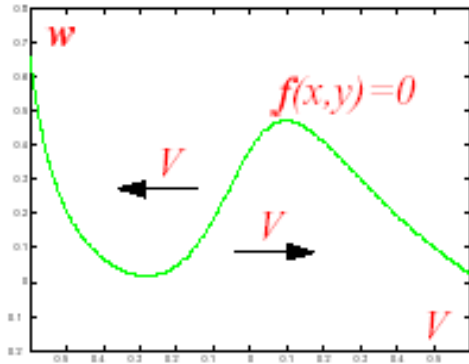
From **mathematical point of view** a system is **excitable** when small **perturbations** near a rest state can cause large **excursions** for the solution before it returns to the rest.

**Systems are excitable** because they are *near bifurcations* from **rest to oscillatory** dynamics.

The **type of bifurcation** determines **excitable properties** and hence **neuro-computational** features of the brain cells. Revealing these features is the most important goal of **mathematical neuroscience**.

The neuron produce spikes periodically when there is a **large amplitude limit cycle attractor**, which may **coexist** with the **quiescent state**.





Most of the bifurcations discussed here can be illustrated using a two-dimensional (planar) system of the form

$$\mu \cdot x' = f(x, y)$$

$$y' = g(x, y)$$

Much insight into the behavior of such systems can be gained by considering their nullclines.

the sets determined by the conditions  $f(x, y) = 0$  or  $g(x, y) = 0$ .

When  $0 < \mu \ll 1$  nullclines are called fast and slow, respectively. Since the language of nullclines is universal in many areas of applied mathematics

# Bursters

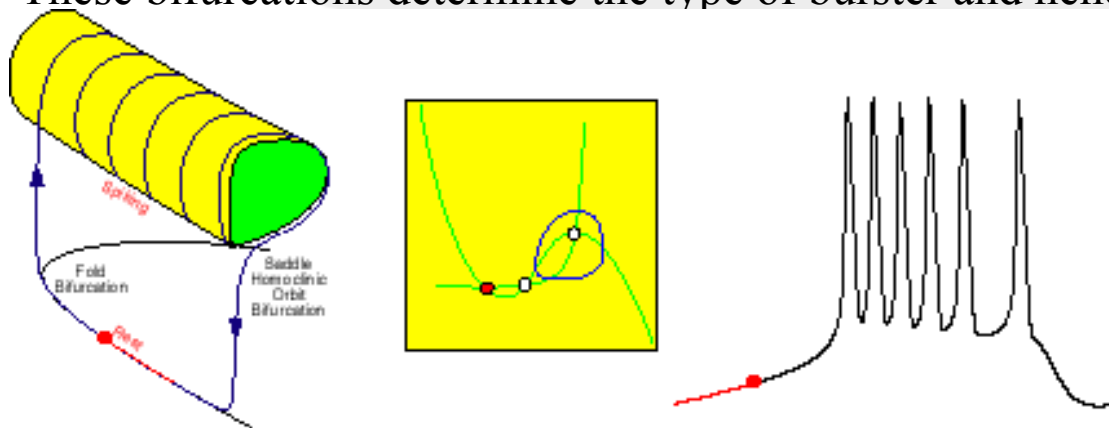
When neuron activity **alternates** between a **quiescent state** and **repetitive spiking**, the neuron activity is said to be **bursting**. It is usually caused by a slow voltage- or calcium-dependent process that can modulate fast spiking activity.

There are **two important bifurcations** associated with bursting:

*Bifurcation of a quiescent state that leads to repetitive spiking.*

*Bifurcation of a spiking attractor that leads to quiescence.*

These bifurcations determine the type of burster and hence its neuro-computational features.



# Modelling Neural Dynamics

Usually they are express in form of ODEs (Ordinary Differential Equations)

## INTEGRATE-AND-FIRE

$$v' = I + a - bv \quad \text{if } v \geq v_{thresold} \quad \text{then } v \leftarrow c$$

$I$   $\longrightarrow$  Input Current  
 $v$   $\longrightarrow$  Membrane Potential  
 $c$   $\longrightarrow$  Reset Value

FLOPS

5 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos	
-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-

## IF WITH ADAPTATION

$$v' = I + a - bv + g(d - v)$$

$$g' = (e\delta(t) - g) / \tau$$

$g$   $\longrightarrow$  Conductance  
 $\delta$   $\longrightarrow$  Dirac

FLOPS

10 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos	
-	+	-	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-

# Modelling Neural Dynamics

## QUADRATIC IF (Ermentrout-Koppel)

$$v' = I + a(v - v_{rest})(v - v_{threshold}) \quad \text{se } v = v_{peak} \quad \text{allora } v \leftarrow c$$

$v_{rest}$   $\longrightarrow$  Threshold  
 $v_{threshold}$   $\longrightarrow$  V Threshold

FLOPS 7 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	-	-	-	-	-	+	-	-	+	-	-	+	+	-	-	-	-	-

## IF OR BURST

$$v' = I + a - bv + gH(v - v_h)h(v_T - v) \quad \text{se } v = v_{threshold} \quad \text{allora } v \leftarrow c$$

$$h' = \begin{cases} -h/\tau^- & \text{se } v > v_h \\ (1-h)/\tau^+ & \text{se } v < v_h \end{cases}$$

$H$   $\longrightarrow$  Heaviside Function

$h$   $\longrightarrow$  T-current function

FLOPS 13 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	?	+	-	-	-	-	-	+	+	+	-	+	+	-	-	-	?

## RESONATE-AND-FIRE

$$z' = I + (b + i\omega)z \quad \text{se } \text{Im}z \geq a_{threshold} \quad \text{allora } z \leftarrow z_0(z)$$

$z$   $\longrightarrow$  Membrane Potential  
 $z_0(z)$   $\longrightarrow$  Reset Value

FLOPS 10 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	-	-	-	-	-	+	+	+	+	-	-	+	+	+	-	-	+

## FITZHUGH-NAGUMO

$$v' = a + bv + cv^2 + dv^3 - u$$

$$u' = \varepsilon(ev - u)$$

$u$   $\longrightarrow$  Recovery variable

FLOPS 72 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	-	?	-	-	+	+	+	-	+	-	+	+	-	+	+	-	-

# Modelling Neural Dynamics

## HINDMARSH-ROSE

$$v' = u - F(v) + I - w$$

$$u' = G(v) - u$$

$$w' = (H(v) - w) / \tau$$

FLOPS

120 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	?	?	+	+	+	+	+	+	+	+	+	+	+	+	?	+

## MORRIS-LECAR

$$C v' = -g_L(v - v_L) - g_{Ca} m_\infty (v - v_{Ca}) - g_K n (v - v_K) + I$$

$$n' = \frac{n_\infty - n}{\tau_n}$$

FLOPS

600 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
+	+	+	-	?	-	-	+	+	+	+	+	?	+	+	-	+	+	-	-

## MODELLO POLINOMIALE (Wilson)

$$C v' = -m_\infty (v - 0.5) - 26 u (v + 0.95) - g_T T (v - 1.2) - g_H H (v + 0.95) + I$$

$$u' = \frac{1}{\tau_u} (-u + u_\infty(v))$$

$$T' = \frac{1}{14} (-T + T_\infty(v))$$

$$H' = \frac{1}{45} (-H + 3T)$$

FLOPS

180 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	?	?	+	+	+	+	+	+	+	+	?	+	+	?	?	?

## HODGKIN-HUXLEY

$$C v' = -g_K n^4 (v - v_K) - g_{Na} m^3 h (v - v_{Na}) - g_l (v - v_l) + I$$

$$m' = \alpha_m (1 - m) - \beta_m m$$

$$n' = \alpha_n (1 - n) - \beta_n n$$

$$h' = \alpha_h (1 - h) - \beta_h h$$

FLOPS

1200 for 1 ms

Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
+	+	+	+	?	?	+	+	+	+	+	+	+	+	+	+	+	+	?	+

# Modelling Neural Dynamics

## IZHIKEVICH

$$v' = 0.04v^2 + 5v + 140 - u + I$$

$$u' = a(bv - u)$$

$$\text{If } v \geq +30 \text{ mV, Then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases}$$

$v$   $\longrightarrow$  Membrane Potential

$u$   $\longrightarrow$  Recovery

FLOPS

13 for 1 ms

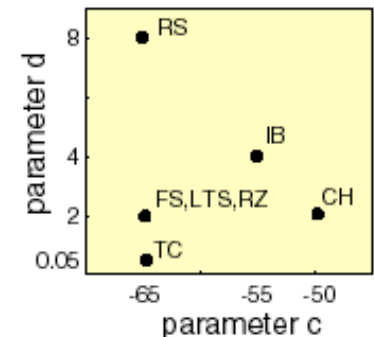
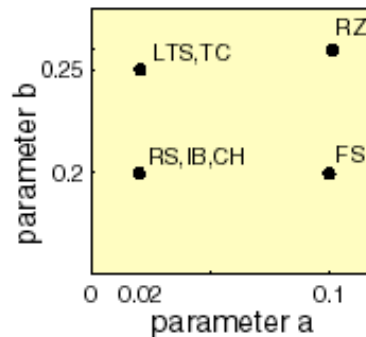
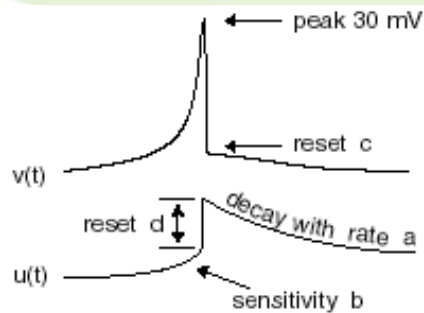
Bio mean	Ton sp	Ph sp	Ton bur	Ph bur	Mix md	frq ad	Sp lat	Sub osc	Res	Integ	Reb sp	Reb bur	Th var	Bist	DAP	Acc	Inib sp	Inib bur	chaos
-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

# Izhikevich Model

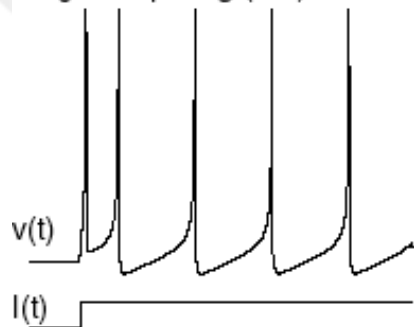
$$v' = 0.04v^2 + 5v + 140 - u + I$$

$$u' = a(bv - u)$$

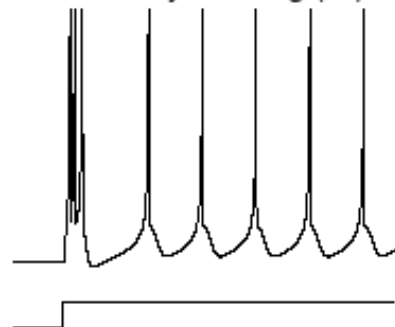
if  $v = 30$  mV,  
then  $v \leftarrow c$ ,  $u \leftarrow u + d$



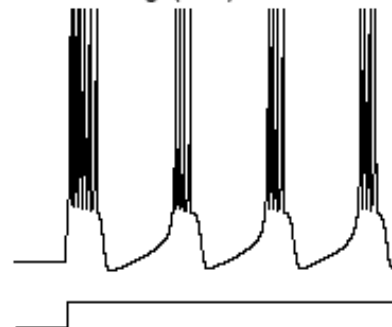
regular spiking (RS)



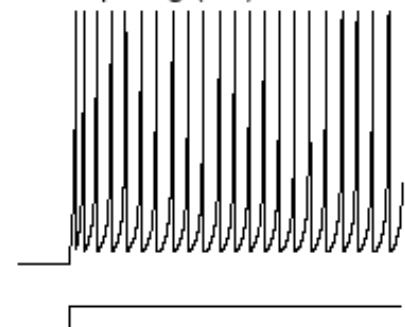
intrinsically bursting (IB)



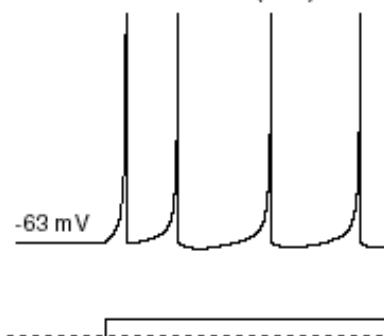
chattering (CH)



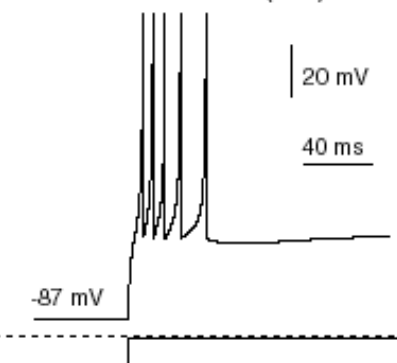
fast spiking (FS)



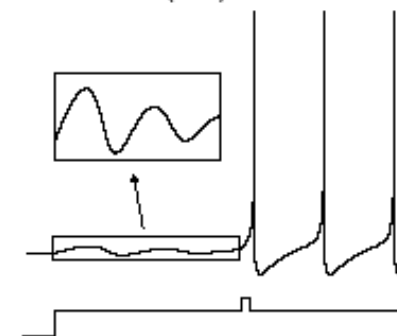
thalamo-cortical (TC)



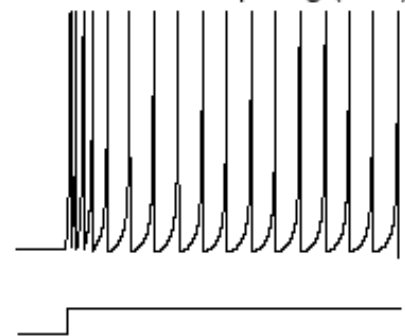
thalamo-cortical (TC)



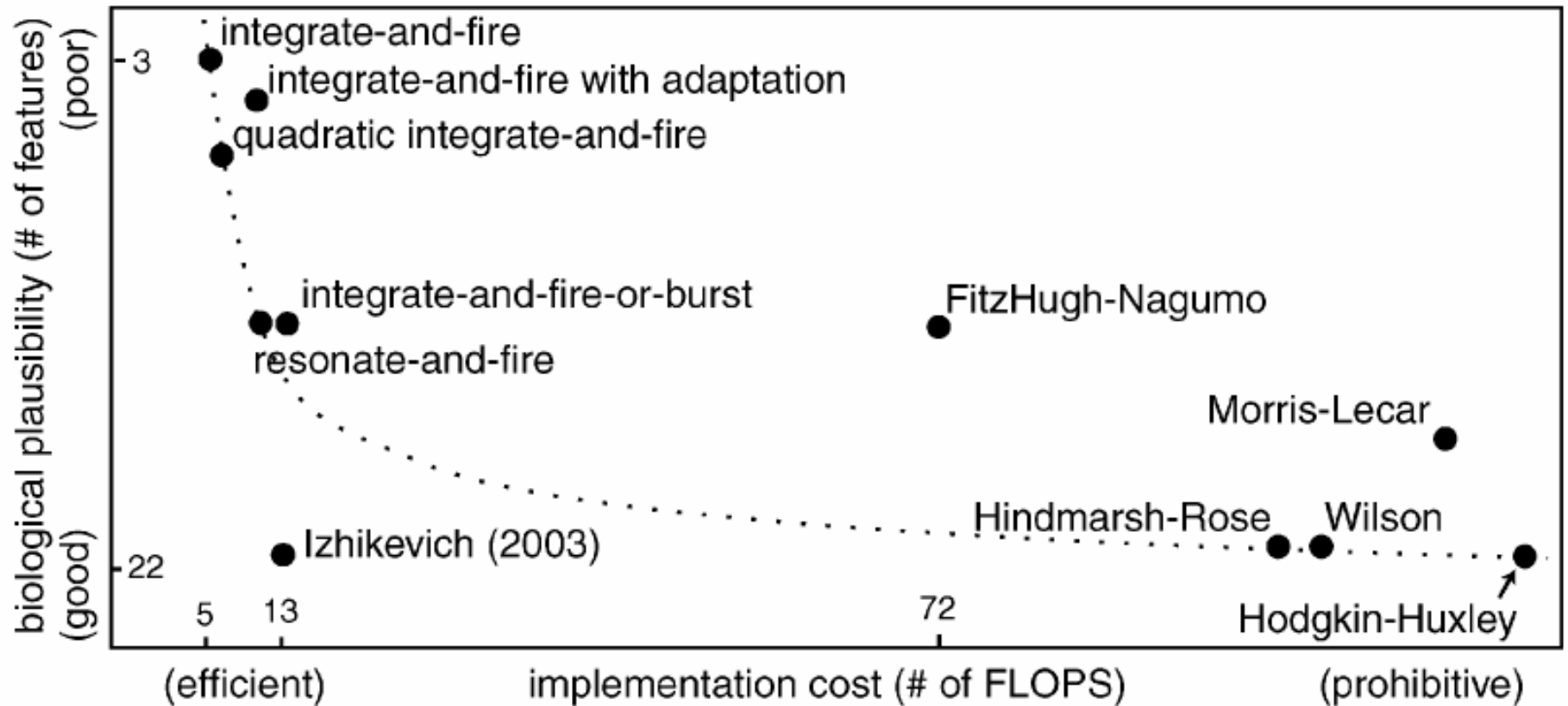
resonator (RZ)



low-threshold spiking (LTS)





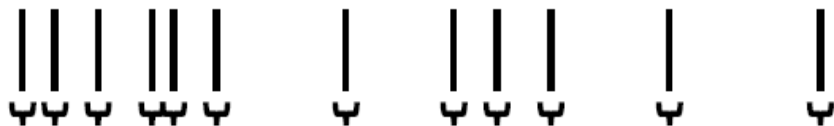


# The Neural Code

## Spike trains



## Where is the Information?

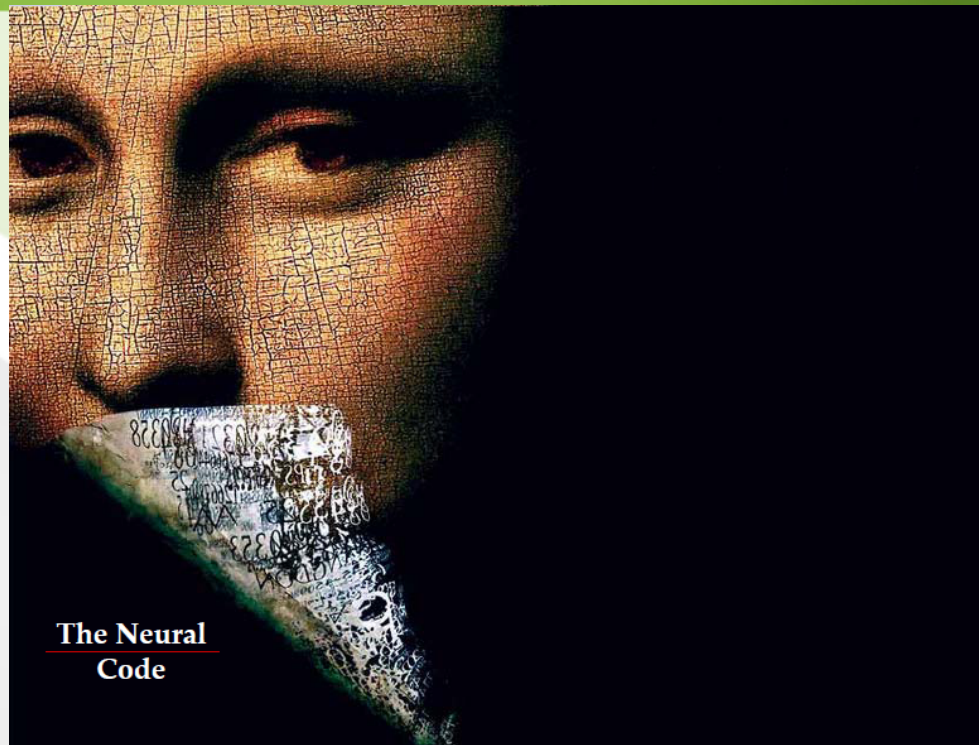


Frequency?

Spikes?



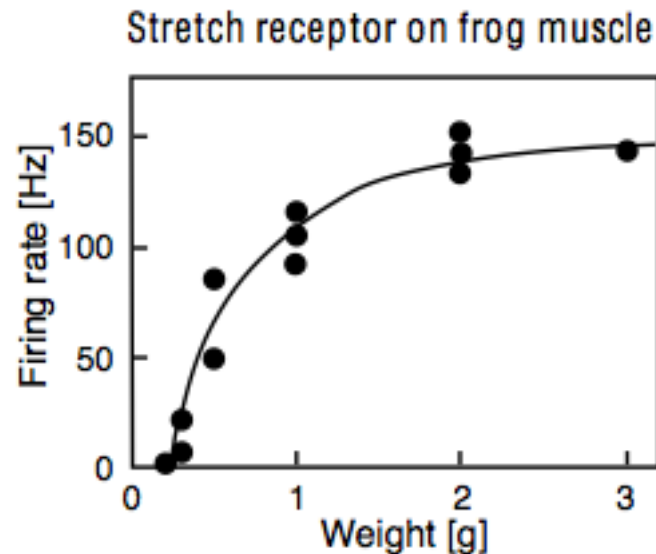
Inter/Intra spike interval?



The Neural  
Code

# The firing rate hypothesis

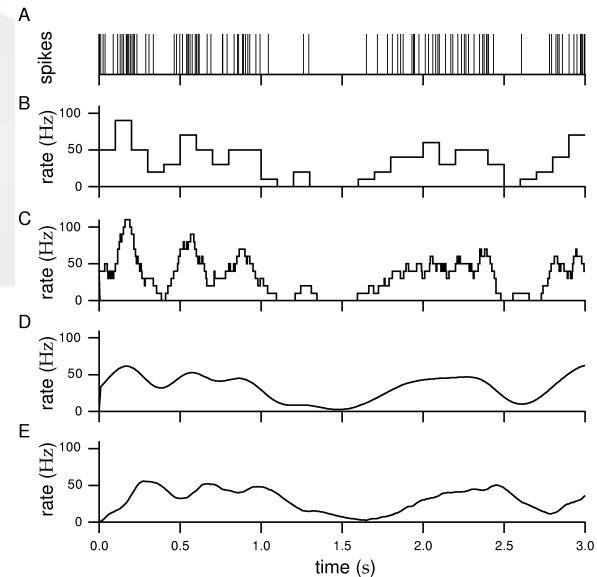
Stimulus features are encoded through the neural firing rate (response curves).



Edgar Adrian  
The Nobel Prize in Physiology or Medicine 1932

Time-dependent firing rate counts number of spikes in a short time interval (averaged over trials):  $r(t) = \frac{1}{\Delta t} \int_t^{t+\Delta t} \rho(\tau) d\tau$ .

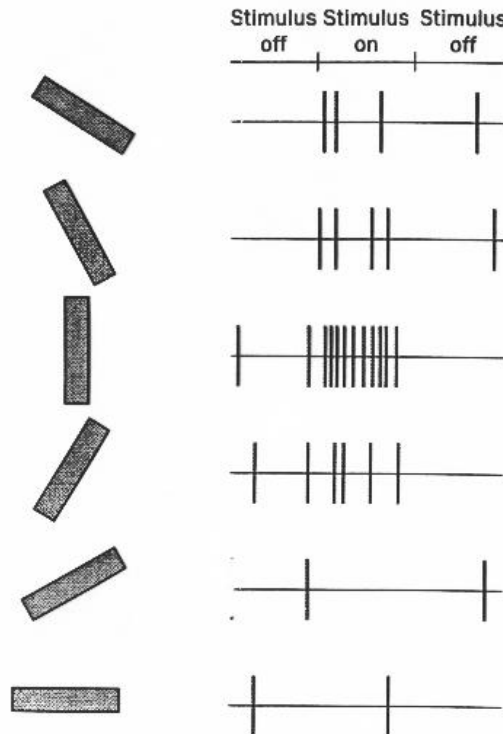
For any  $t > 0$ , each interval contains 0,1 spike.  
Then,  $r(t)$  averaged over trials is  
the probability of any trial firing at time  $t$ .  
B: 100 ms bins



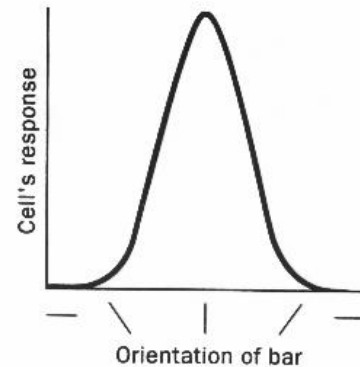
# The firing rate hypothesis

Receptive field: area in the outside/physical world for which a neuron is responsive.

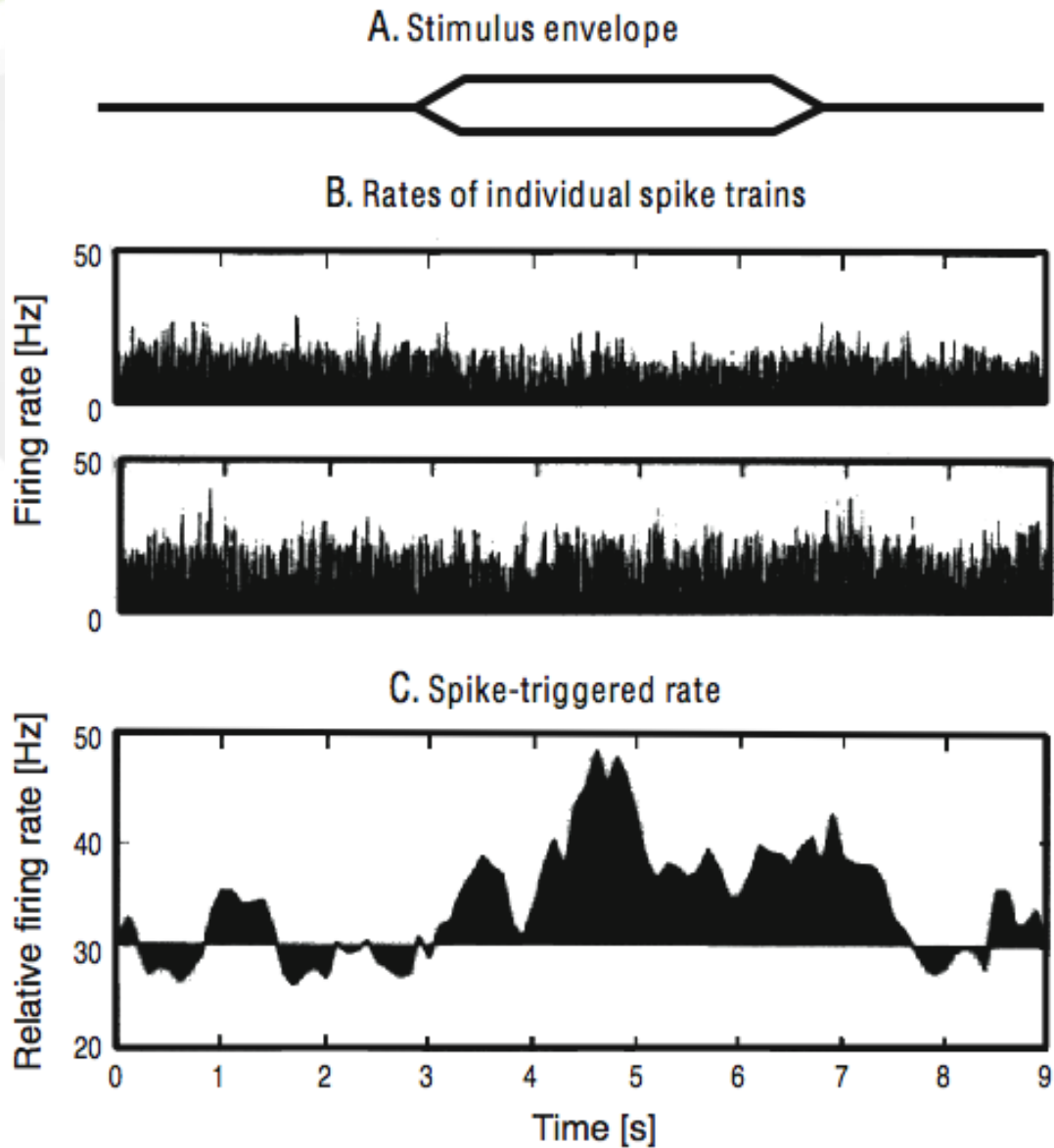
Feature preference



Tuning curve of V1 neuron in cat



# The correlation code hypothesis

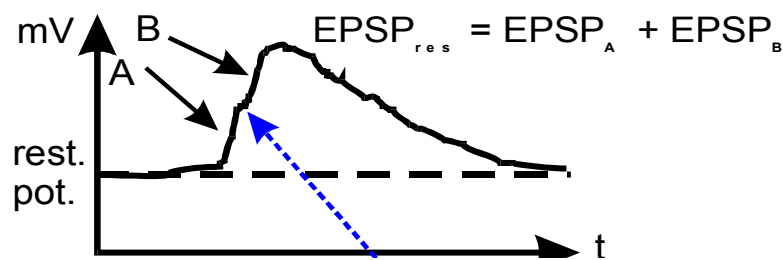
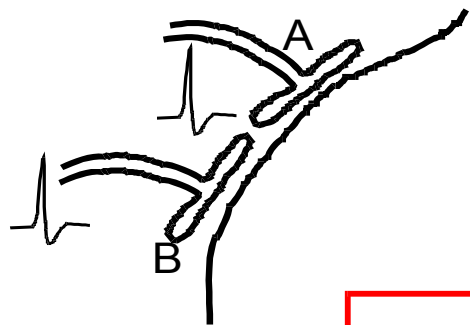
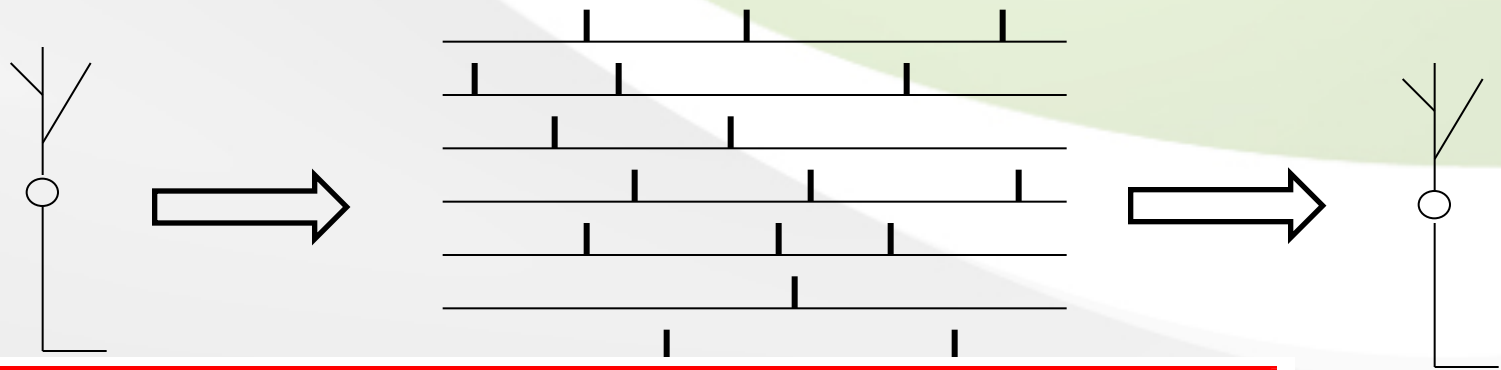


Stimulus features are encoded by neurons firing around the same time

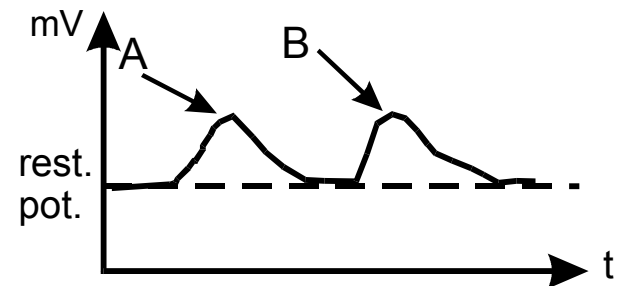
# The Neural Code

Neurons communicate via **exact spike timing**

**Firing rate alone does not carry all the relevant information**



If the difference in arrival times is too large, temporal summation does not occur anymore !



# The Neural Code

**Edelman** (Nobel laureate in Medicine) proposed the **theory of neuronal group selection (TNGS)**, also known as Neural Darwinism,

**Edelman** stated that DNA does not contain all information needed to code all brain connections. DNA provides basic species-related information exclusively.

**Living and dead cells are regulated by stochastic rules, therefore each brain is different from each other.**



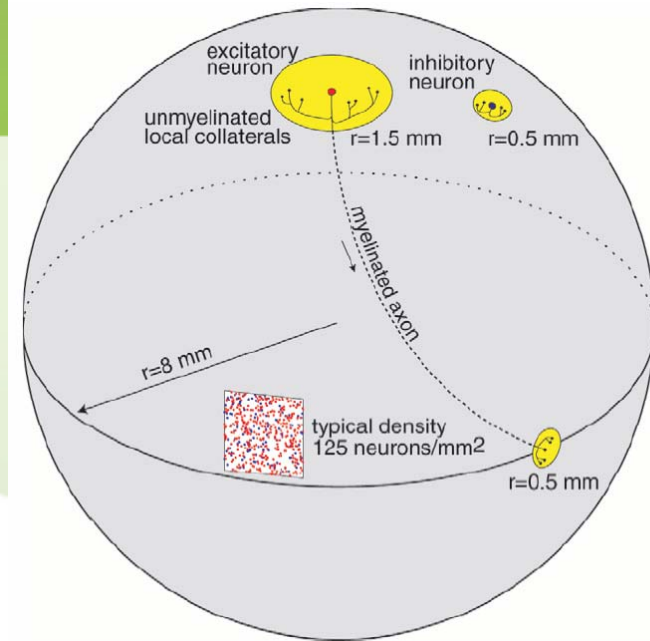
Indeed, in the human brain there are  $10^{11}$  neurons, with  $10^{15}$  synapses. DNA has 109 pairs of nucleotides



# The Neural Code

Neural Groups are characterized by:

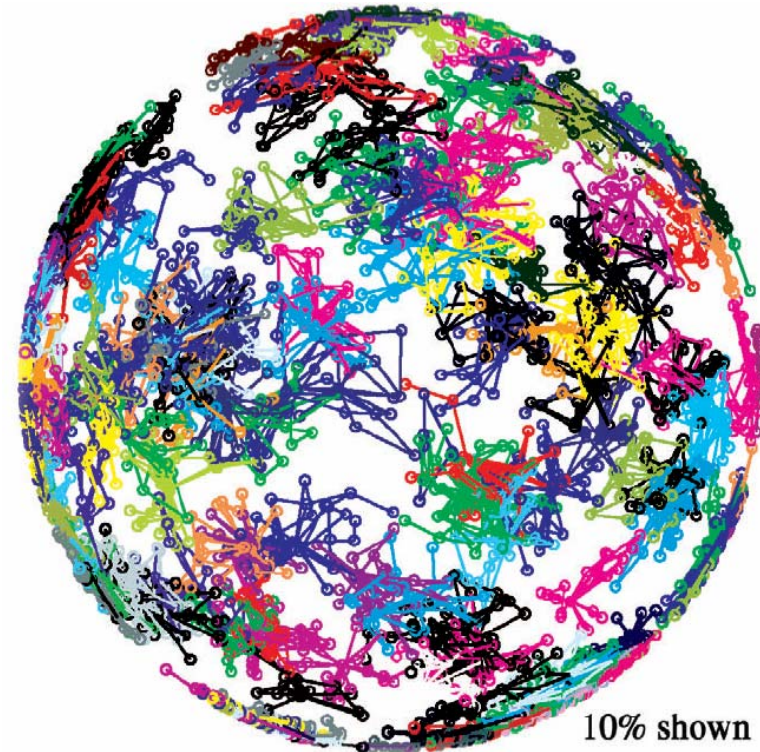
- Biological Selection (DNA)
- Experiential Selection
- **Reentry**



Neural Groups should be considered as the basic processing unit of the brain

How to model Neural Groups in a Spiking Neural Network?

**Time** must be taken into account



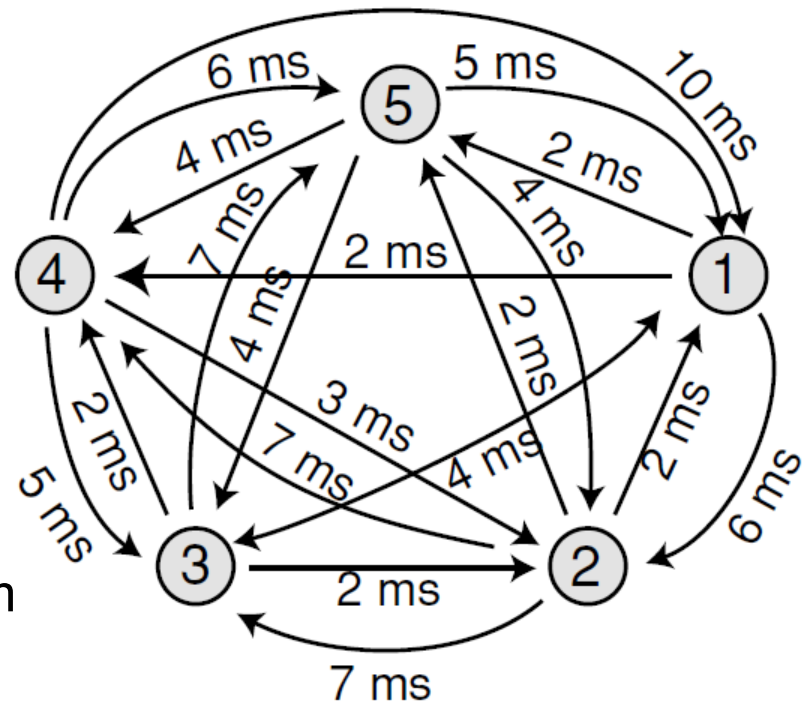
# Spiking neural network

The network consists of cortical spiking neurons with axonal conduction delays and spike timing-dependent plasticity (STDP).

The network is sparse with 0.1 probability of connection between any two neurons.

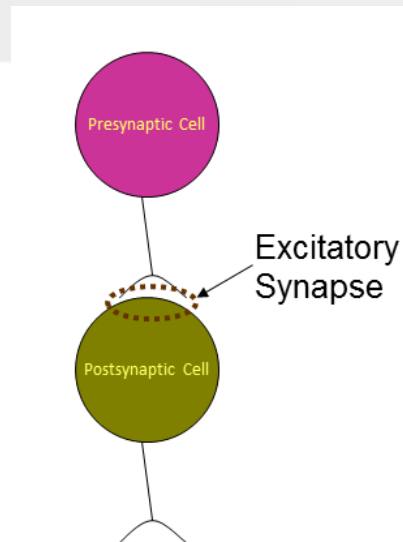
Neurons are connected to each other randomly

Synaptic connections among neurons have fixed conduction delays, which are random integers between 1 ms and 20 ms.



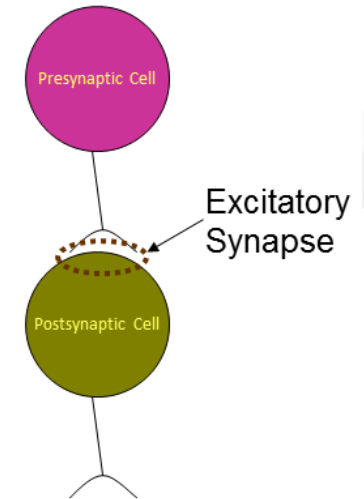
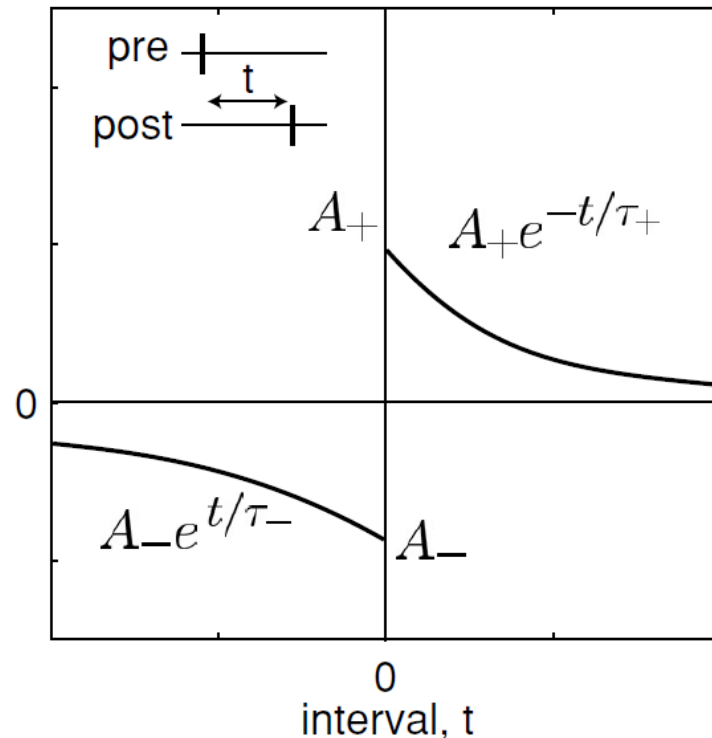
# STDP rule (spike-timing-dependent plasticity)

Initially, all synaptic connections have equal weights.  
The magnitude of change of synaptic weight  
depends on the timing of spikes.



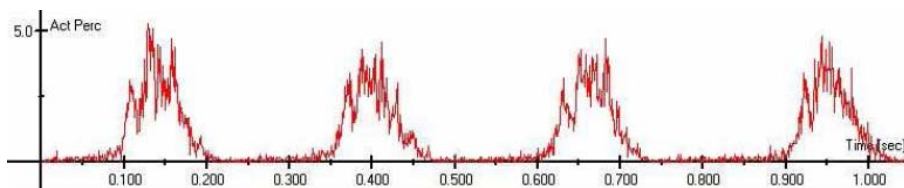
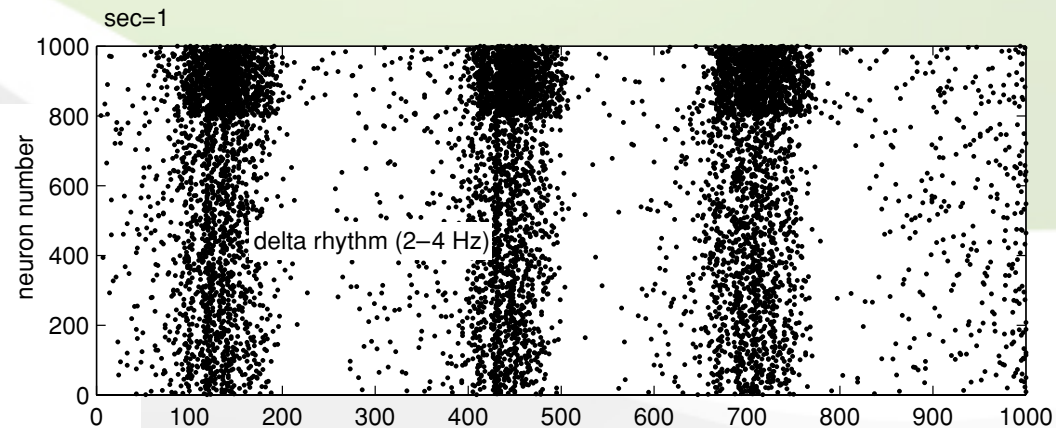
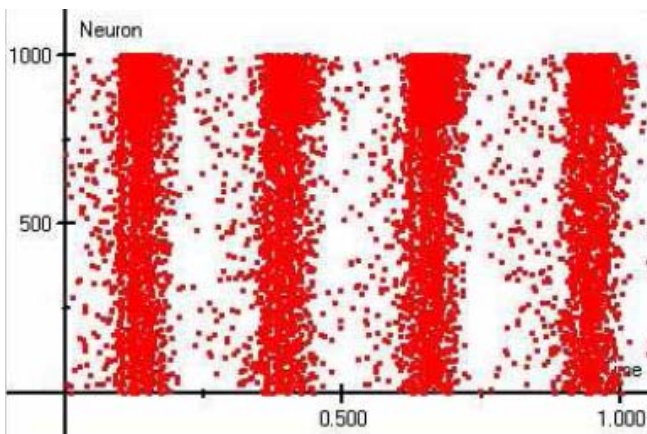
# STDP rule (spike-timing-dependent plasticity)

If the presynaptic spike arrives at the postsynaptic neuron before the postsynaptic neuron fires—for example, it causes the firing—the synapse is potentiated.

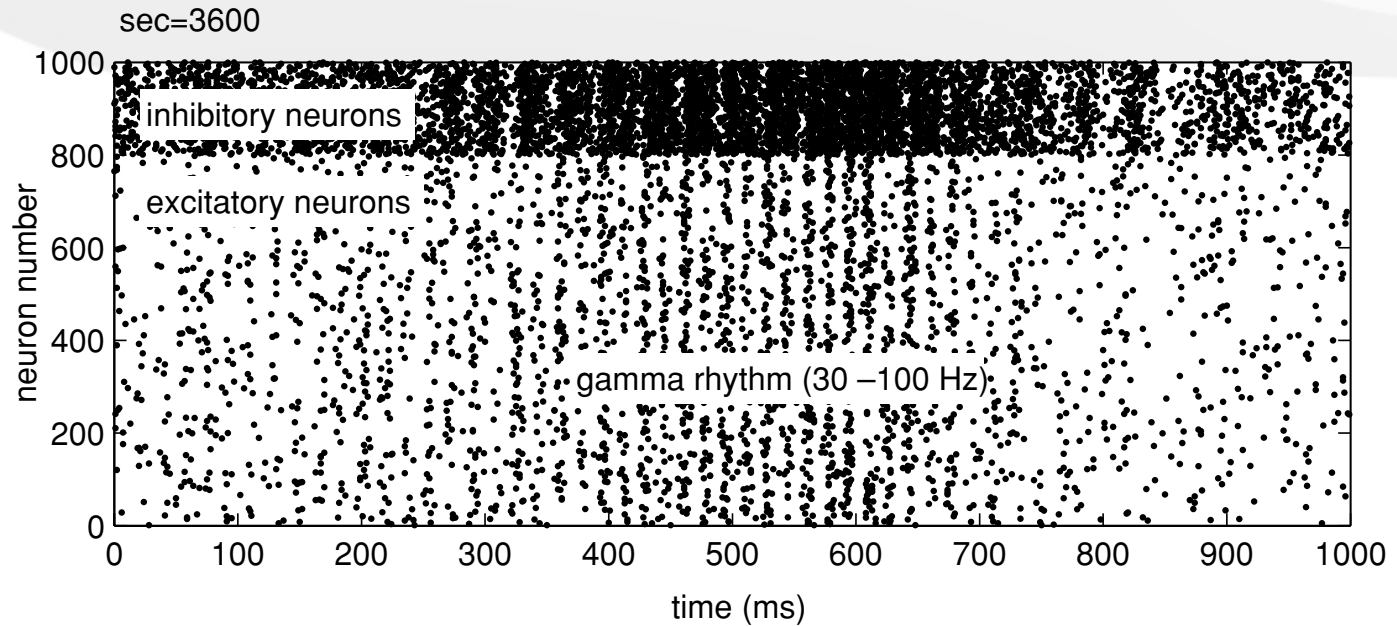
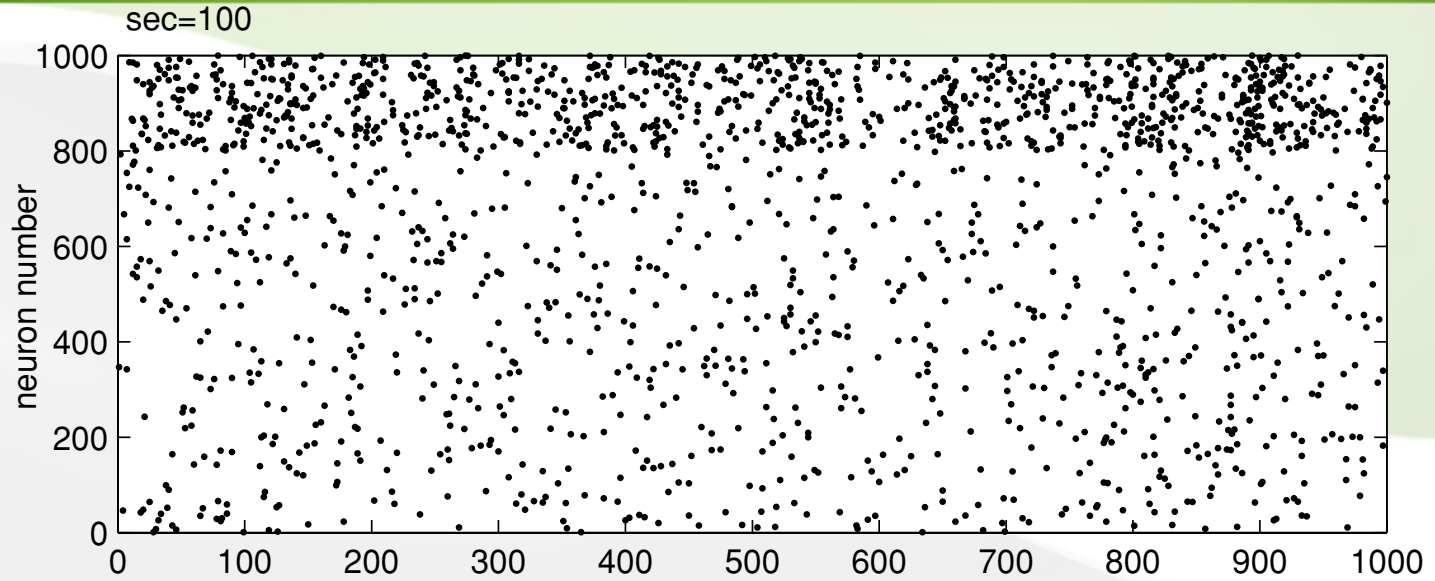


If the presynaptic spike arrives at the postsynaptic neuron after it fired, that is, it brings the news late, the synapse is depressed.

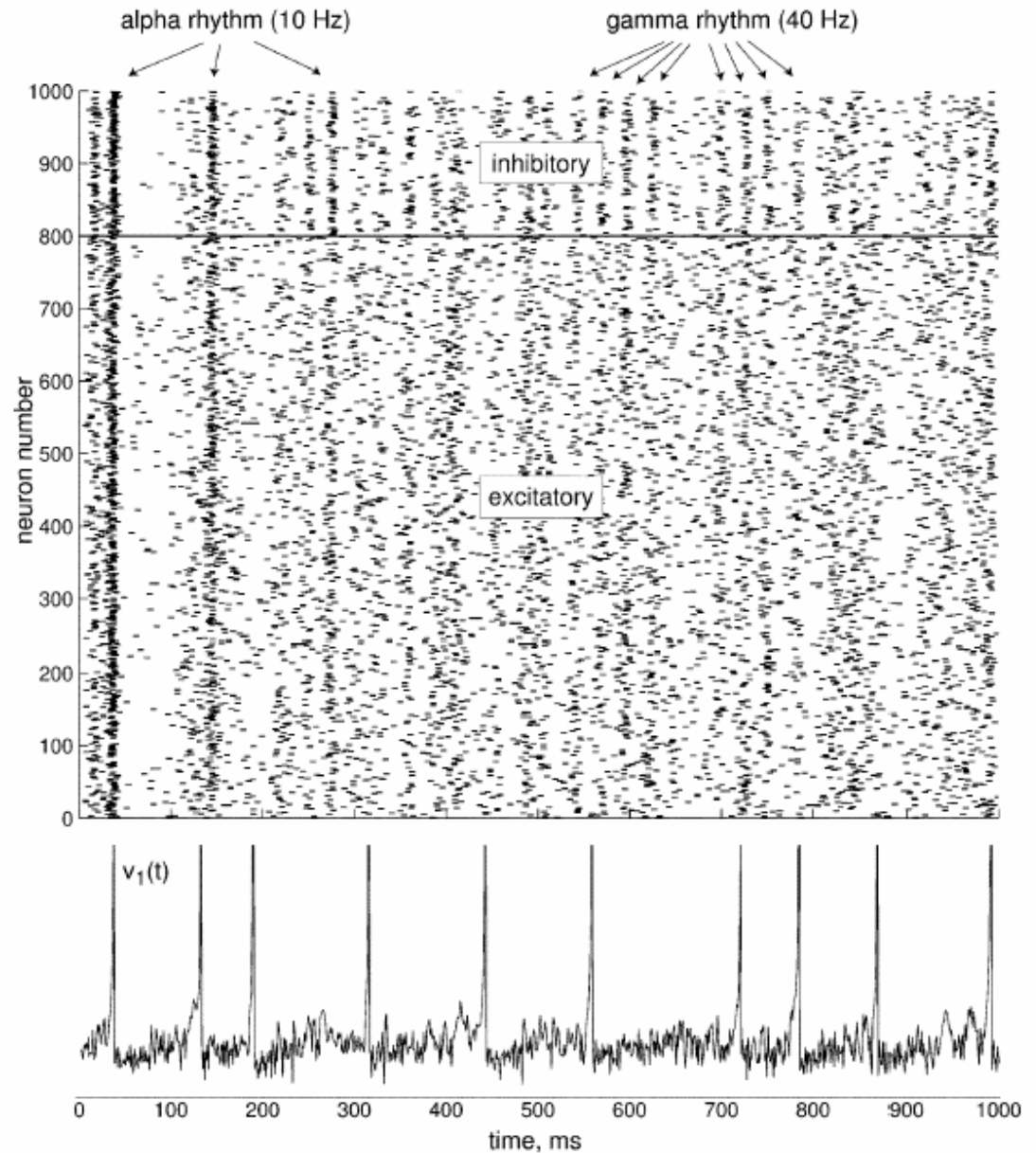
## First Seconds of Simulation



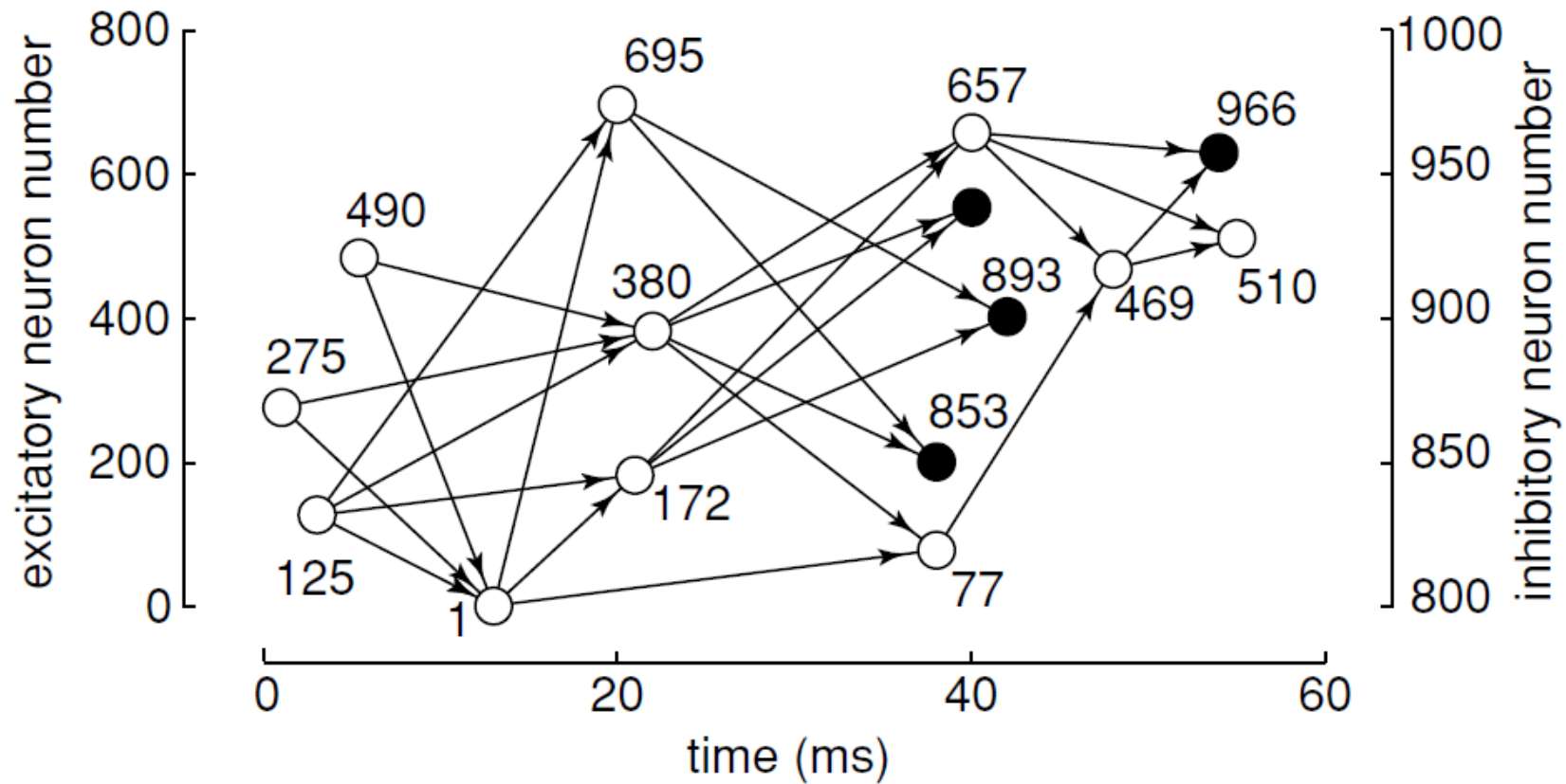
**delta waves (<4 Hz)**



## First Minutes of Simulation



# Polychronous Neural Group (PNG)





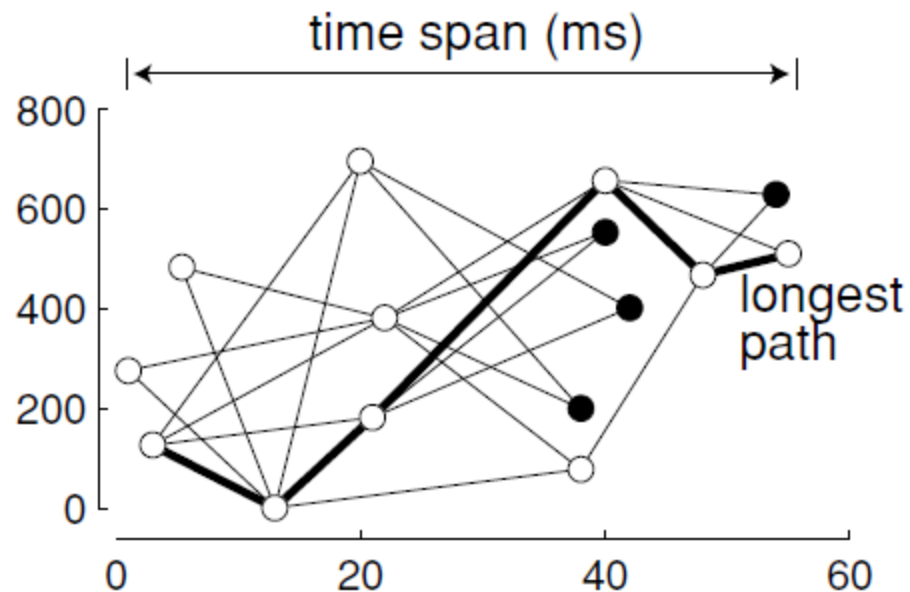
# Characteristics of polychronous groups

The groups have different

Sizes

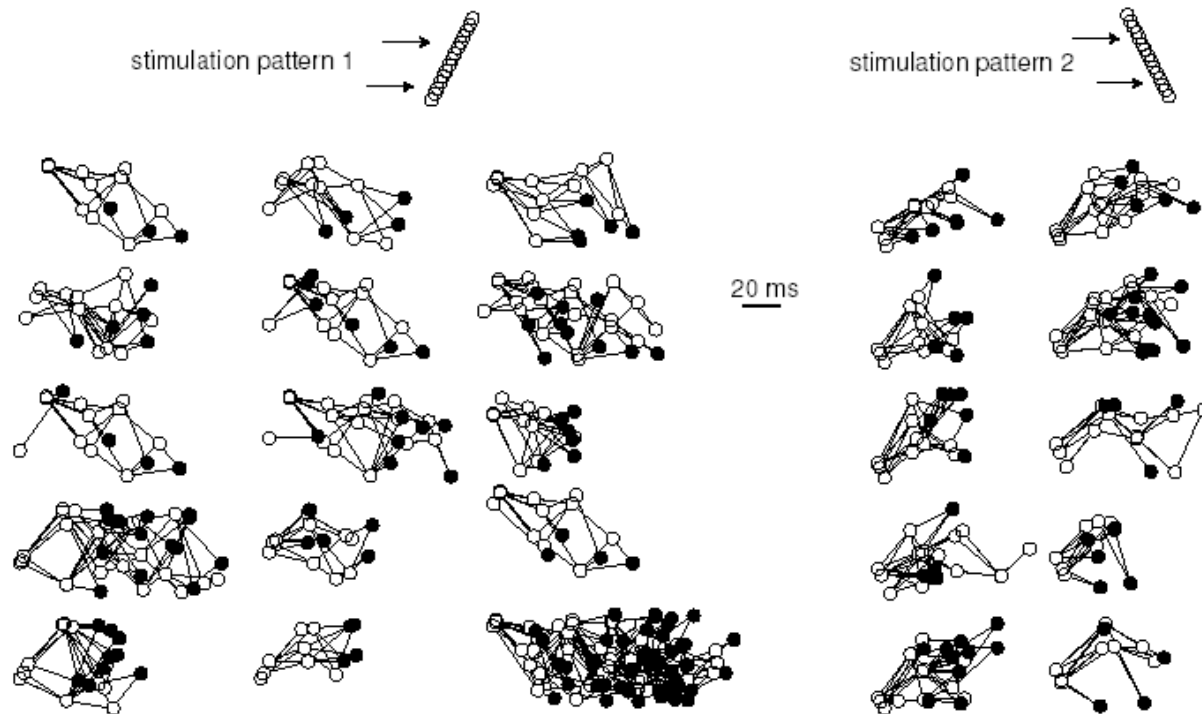
Lengths

Time spans

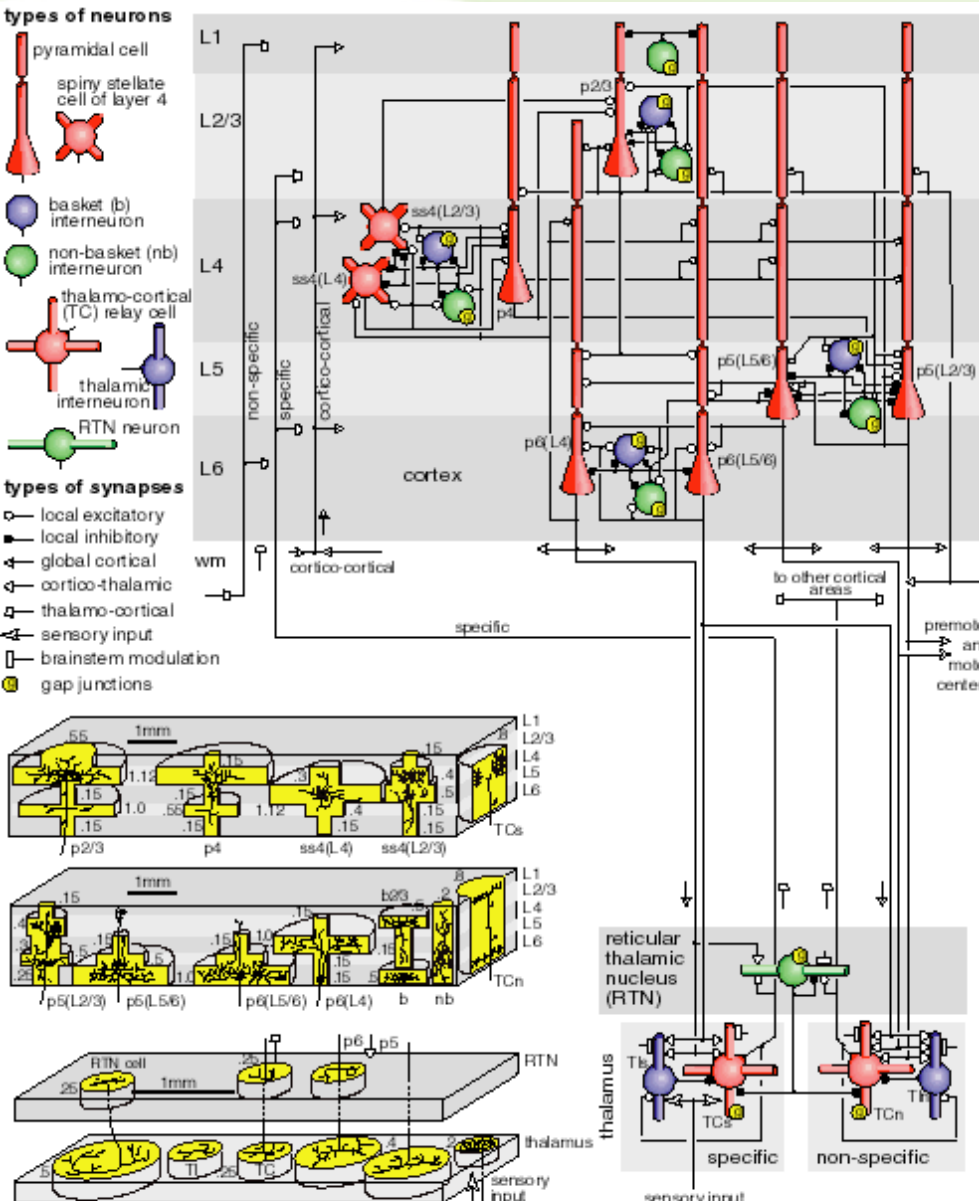


# Representations of Memories and Experience

Persistent stimulation of the network with two **spatio-temporal patterns** result in emergence of polychronous groups that represent the patterns. the **groups activate whenever the patterns are present**.



# Simulation of Large-Scale Brain Models



In 2005 Izhikevich finished simulation of a model that has the size of the human brain. The model has 100,000,000,000 neurons (hundred billion or  $10^{11}$ ) and almost 1,000,000,000,000,000 (one quadrillion or  $10^{15}$ ) synapses.

It represents  $300 \times 300 \text{ mm}^2$  of mammalian thalamo-cortical surface, specific, non-specific, and reticular thalamic nuclei, and spiking neurons with firing properties corresponding to those recorded in the mammalian brain.

The model exhibited alpha and gamma rhythms, moving clusters of neurons in up- and down-states, and other interesting phenomena

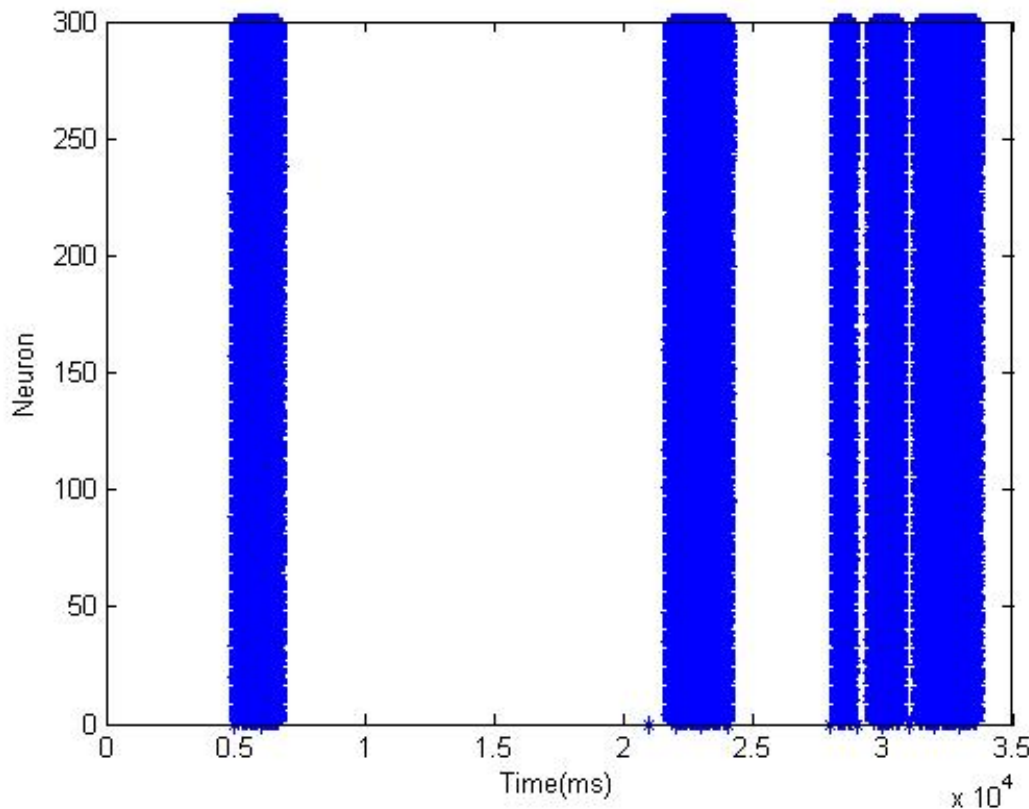
One second of simulation took 50 days on a beowulf cluster of 27 processors (3GHz each).

# A stochastic version of Izhichevich Model

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + \epsilon(\mu, \sigma)$$

$$\frac{du}{dt} = a(bv - u)$$

If  $v \geq 30$   
then  $\{c \rightarrow v, u \rightarrow u+d\}$



Persistent Bursting Activity!

# Cells in the Central Nervous Systems



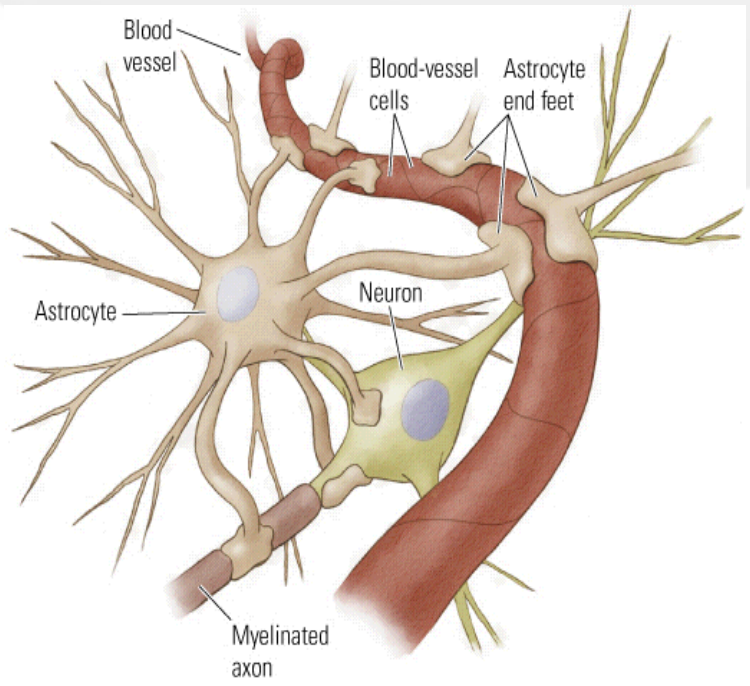
## Neuron

Electrical Activity



## Glia

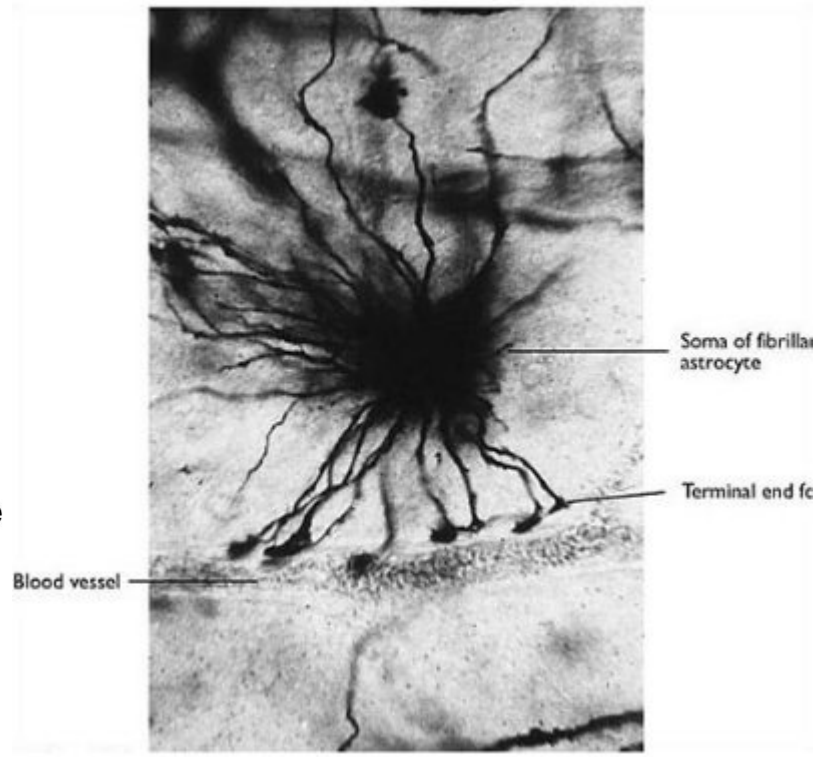
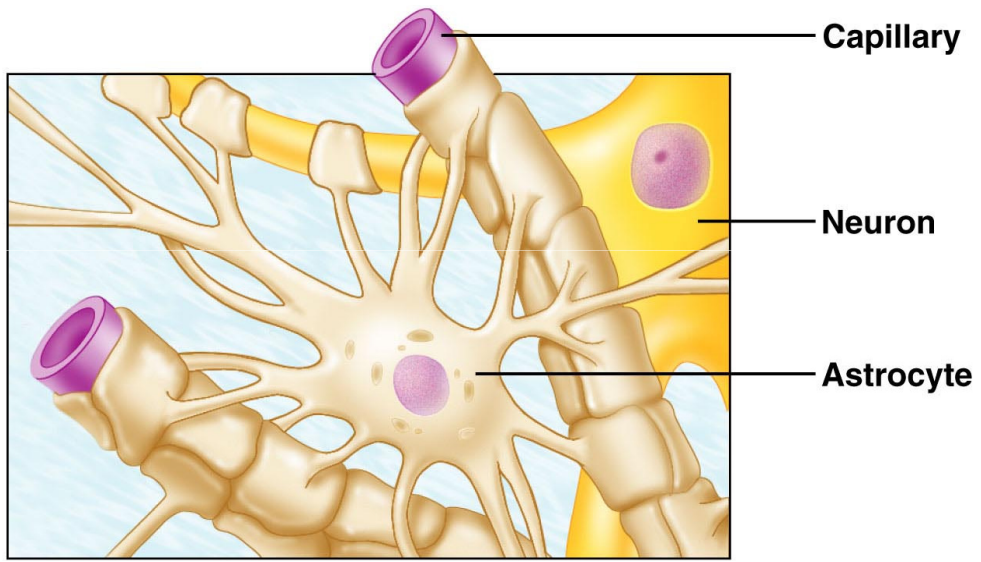
Biochemical Activity



- Astrocyte
- Oligodendrocytes
- Schwann Cell

# Astrocytes.....most abundant glial cell type

Form anatomical link between neurons and arterioles



- Radial astrocytes: surround ventricles
- Protoplasmic astrocytes: in gray matter
- Fibrous astrocytes: in white matter

# Function

	AMPARs	NMDARs	P2XR	Dopamine receptors	GABARs	Glycine receptors	MGluRs	P2YRs
Cortex	+	+	+	-	-	-	+	+
Hippocampus								
GluR cells	+	-	-	-	+	-	?	+
GluT cells	-	-	-	-	?	-	+	+
Cerebellum	+	-	-	-	+	-	+	+
Basal ganglia	?	-	-	+	-	-	?	?
Spinal cord	+	+	-	-	-	+	+	+

Development

Structural

BBB

Metabolic support

Homeostasis

Signal

**(Before ~1990) Neurons are the only carriers of information in the brain.**

Glia cells exist only for metabolic support



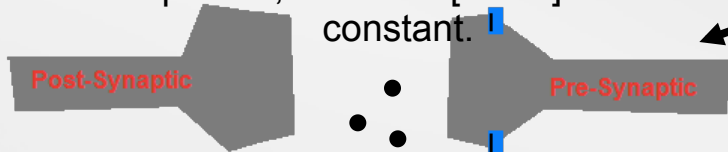
For many years it was thought that process of synaptogenesis, maintenance, and elimination of synaptic contacts was solely neural responsibility



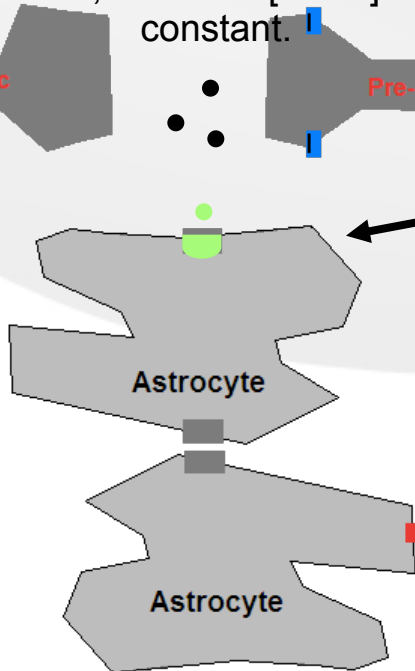
# Glutamate-dependent Astrocyte Modulation of Synaptic Transmission Between Cultured Hippocampal Neurons

To confirm that the increase in neuronal  $Ca^{++}$  was due to an astrocytic-dependant pathway (in contrast to synaptic), Parpura introduced a mGluR antagonist, d-glutamylglycine, into the cell co-culture. As expected, neuronal  $[Ca^{++}]$  remained

Parpura measured neuronal  $[Ca^{++}]$  after Bradykinin injection, and found that  $Ca^{++}$  waves in astrocytes induced a neural  $Ca^{++}$  rise. This leads to a greater potential for synaptic activity.



constant.



When  $[Ca^{++}]$  rises in astrocytes adjacent to the co-cultured neurons, glutamate is released (through exocytosis) and binds to ionotropic glutamate receptors on the neural membrane. This opens  $Ca^{++}$  ion channels to, and extrasynaptic  $Ca^{++}$  flows into the neuron.

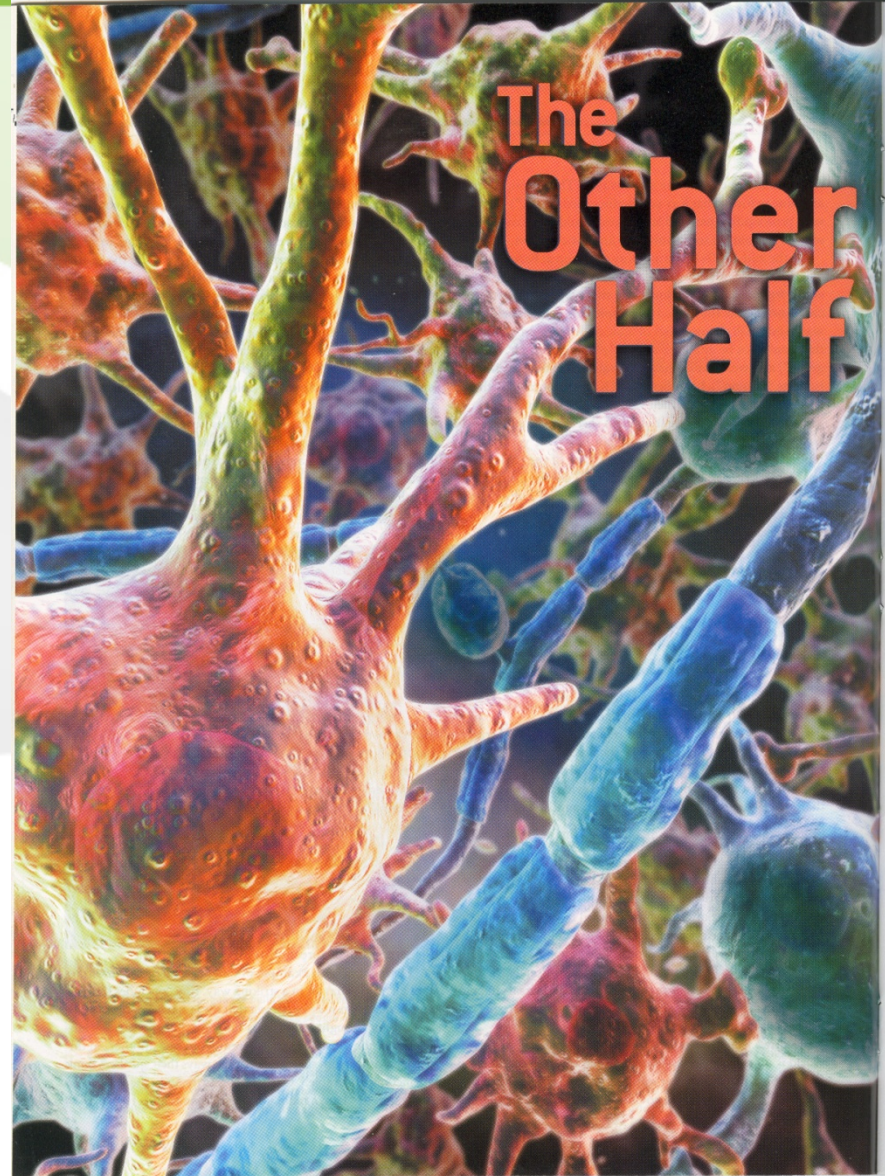
Parpura et al. introduced Bradykinin, an exogenous neuro-ligand, into the neuron-astrocyte co-culture. This glutamate receptor agonist bound to metabotropic glutamate receptor sites on a distal Astrocyte. Intracellular  $[Ca^{++}]$  rises, eventually propagating into a global wave.

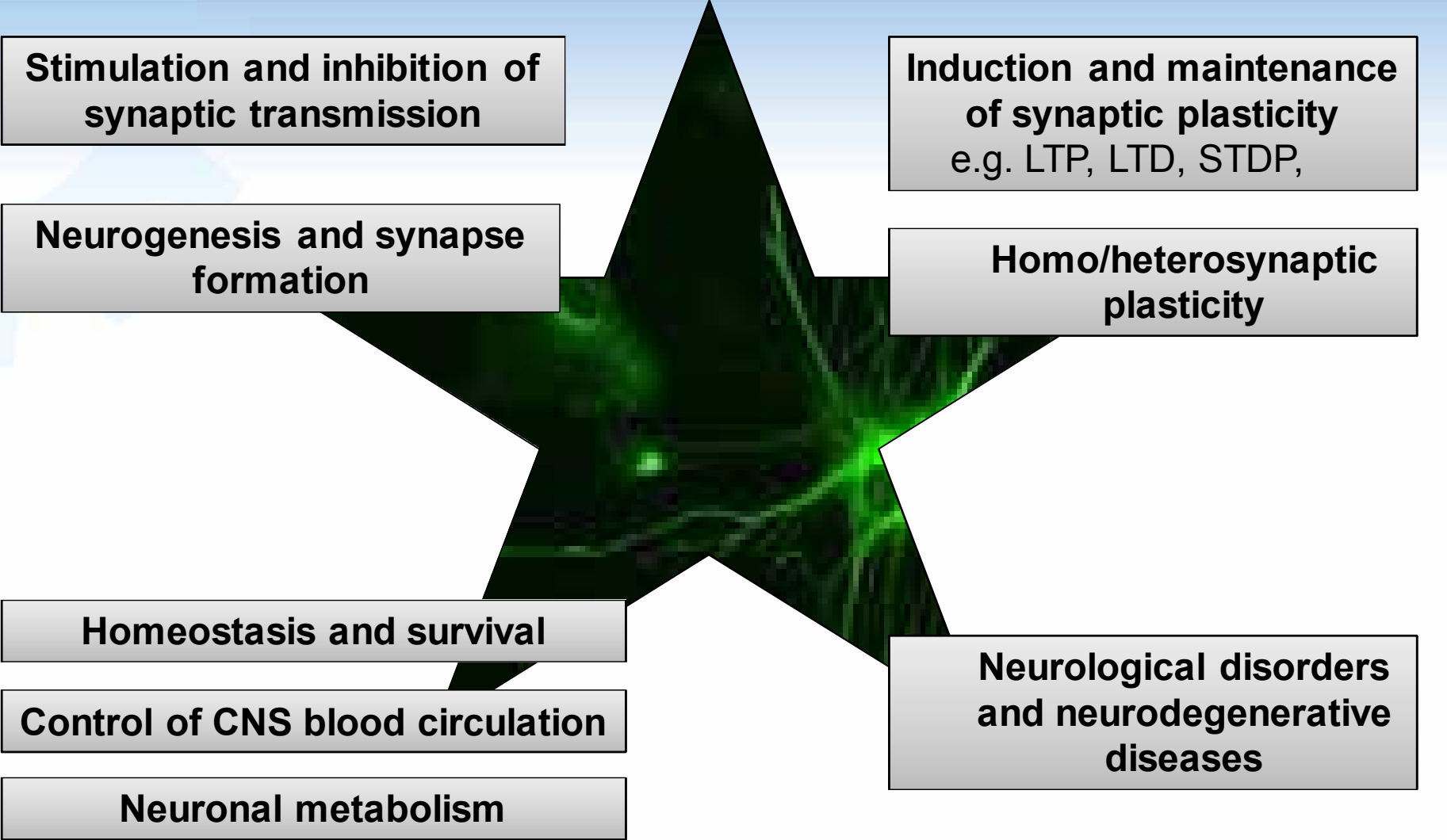
## Before 1990: *Structural support for neurons*

### 1990-2000: *“housekeeping” cells with active support roles*

- Buffering and siphoning of  $[K^+]_{out}$  and  $[Ca^{2+}]_{out}$  after excessive firing
- Uptake of neurotransmitters
  - glutamate (Pellerin and Magistretti, 1994), GABA,
- Release of gliotransmitters
  - glutamate (Parpura et al., 1994), ATP, D-serine, GABA, growth factors, ,  $Ca^{2+}$ -binding buffers (2013)
- Respond to synaptic activity by increasing  $[Ca^{2+}]_i$
- Glutamate-mediated modulation of synaptic transmission
  - Concept of tripartite synapse (Araque et al., 1999).

# Then...the other half of the brain





**Stimulation and inhibition of synaptic transmission**

**Neurogenesis and synapse formation**

**Homeostasis and survival**

**Control of CNS blood circulation**

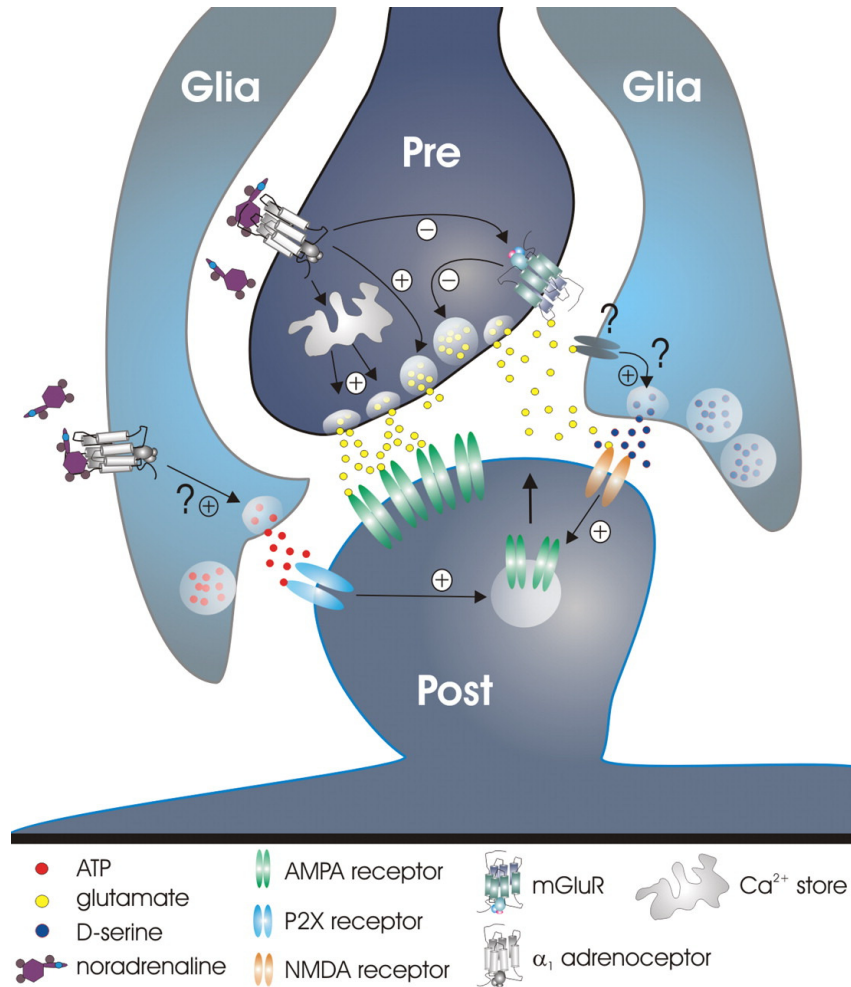
**Neuronal metabolism**

**Induction and maintenance of synaptic plasticity**  
e.g. LTP, LTD, STDP,

**Homo/heterosynaptic plasticity**

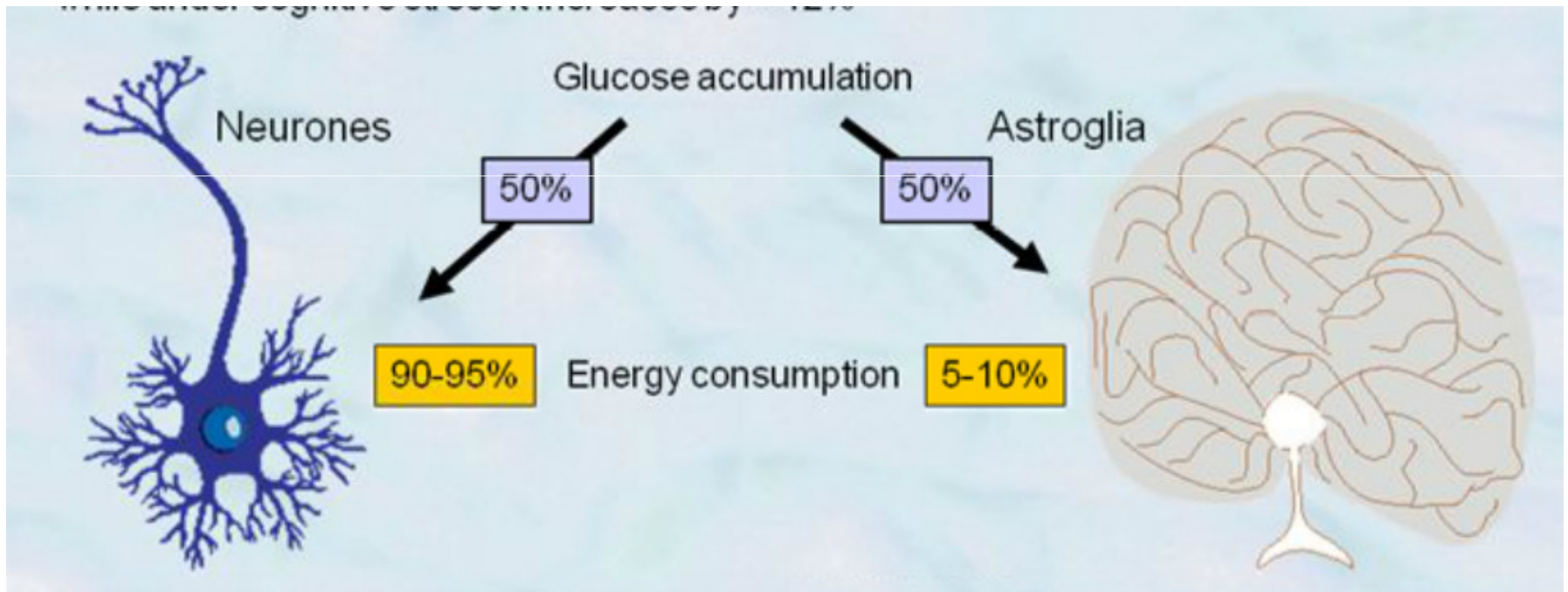
**Neurological disorders and neurodegenerative diseases**

# Tripartate synapse



# Metabolism

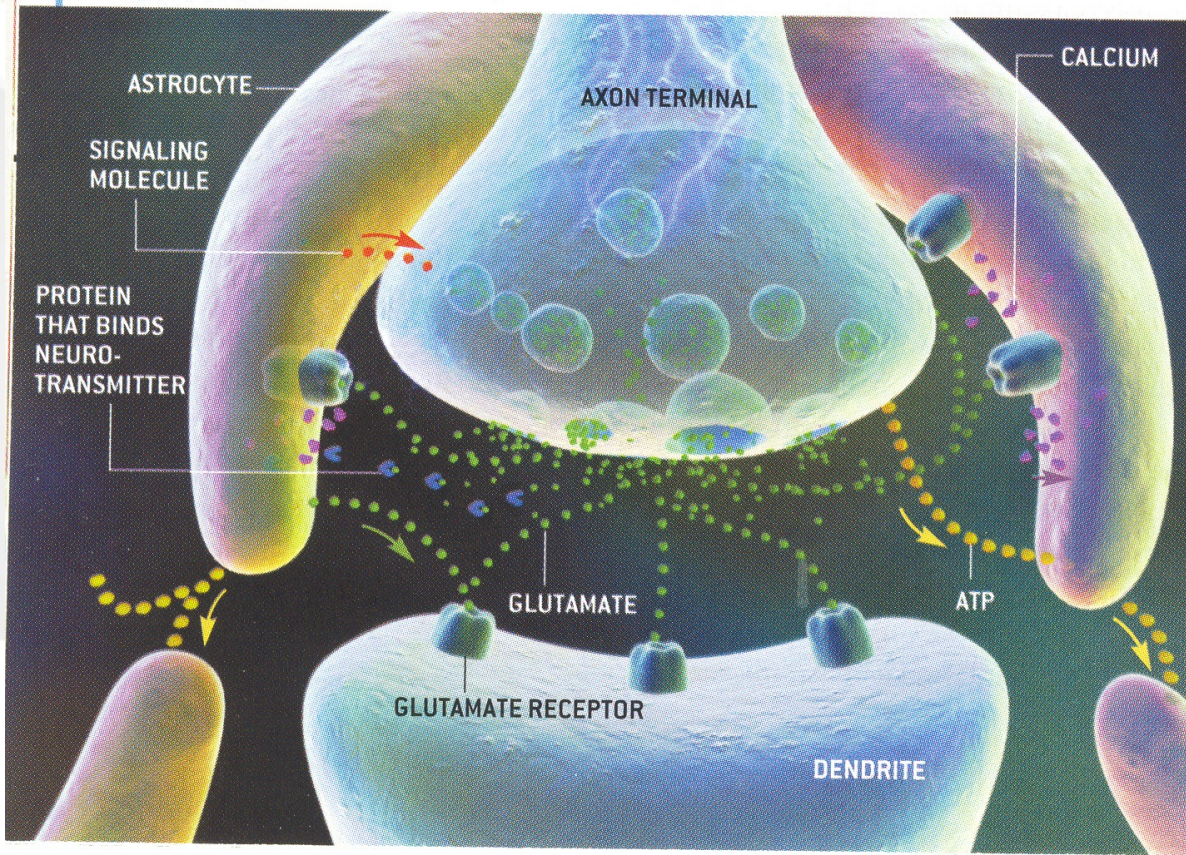
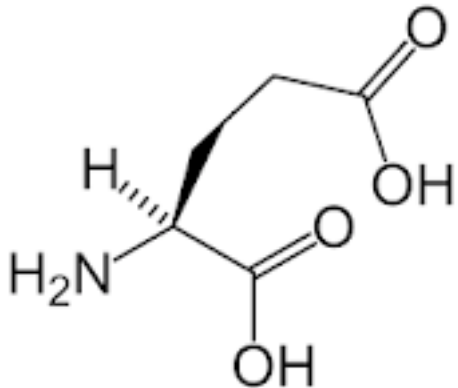
Brain represents approx 2% of total body mass, but consumes 20% of total energy  
-decreases by 40% during sleep  
-increases by 12% under cognitive stress



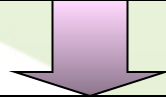
Energy for transmembrane ion gradients

# Glutamate

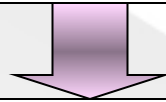
**Glutamate** (the conjugate base of glutamic acid) is abundant in the human body, but particularly in the nervous system and especially prominent in the human brain. It is the brain's main excitatory neurotransmitter



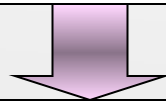
**Glutamate** binds with receptor mGluR



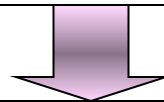
Series of chemical reactions



Production of  $IP_3$  (inositol 1,4,5-trisphosphate)



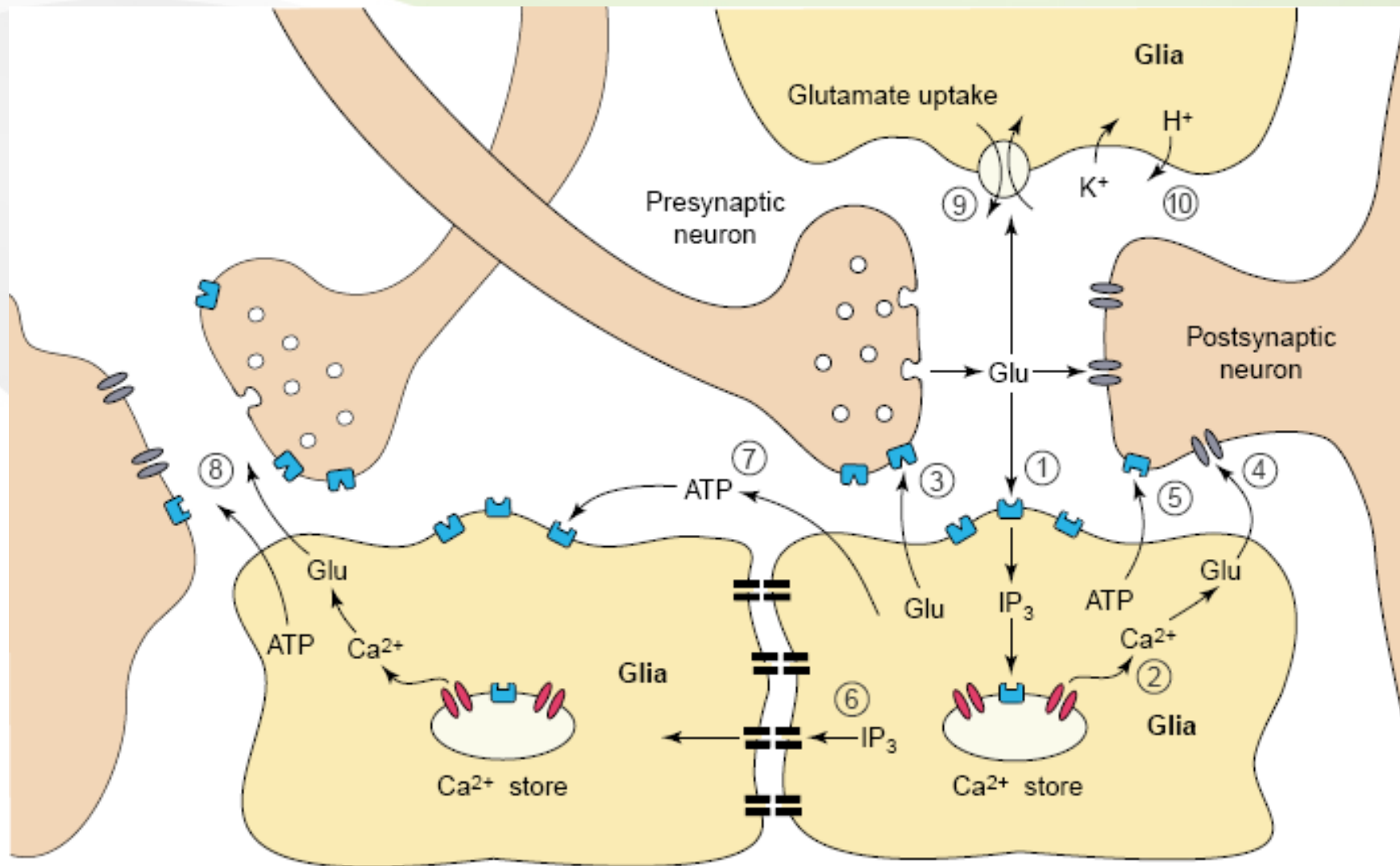
$IP_3$  diffusion through cell cytoplasm



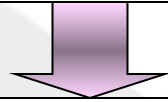
$IP_3$  binds with its receptors in the endoplasmic reticulum



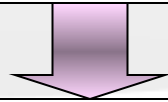
# The tripartite synapse



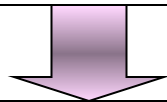
IP<sub>3</sub> receptors release Ca<sup>2+</sup> in the endoplasmic reticulum



Ca<sup>2+</sup> in the endoplasmic reticulum create a gradient of Ca<sup>2+</sup> concentration between the endoplasmic reticulum and the cell cytoplasm

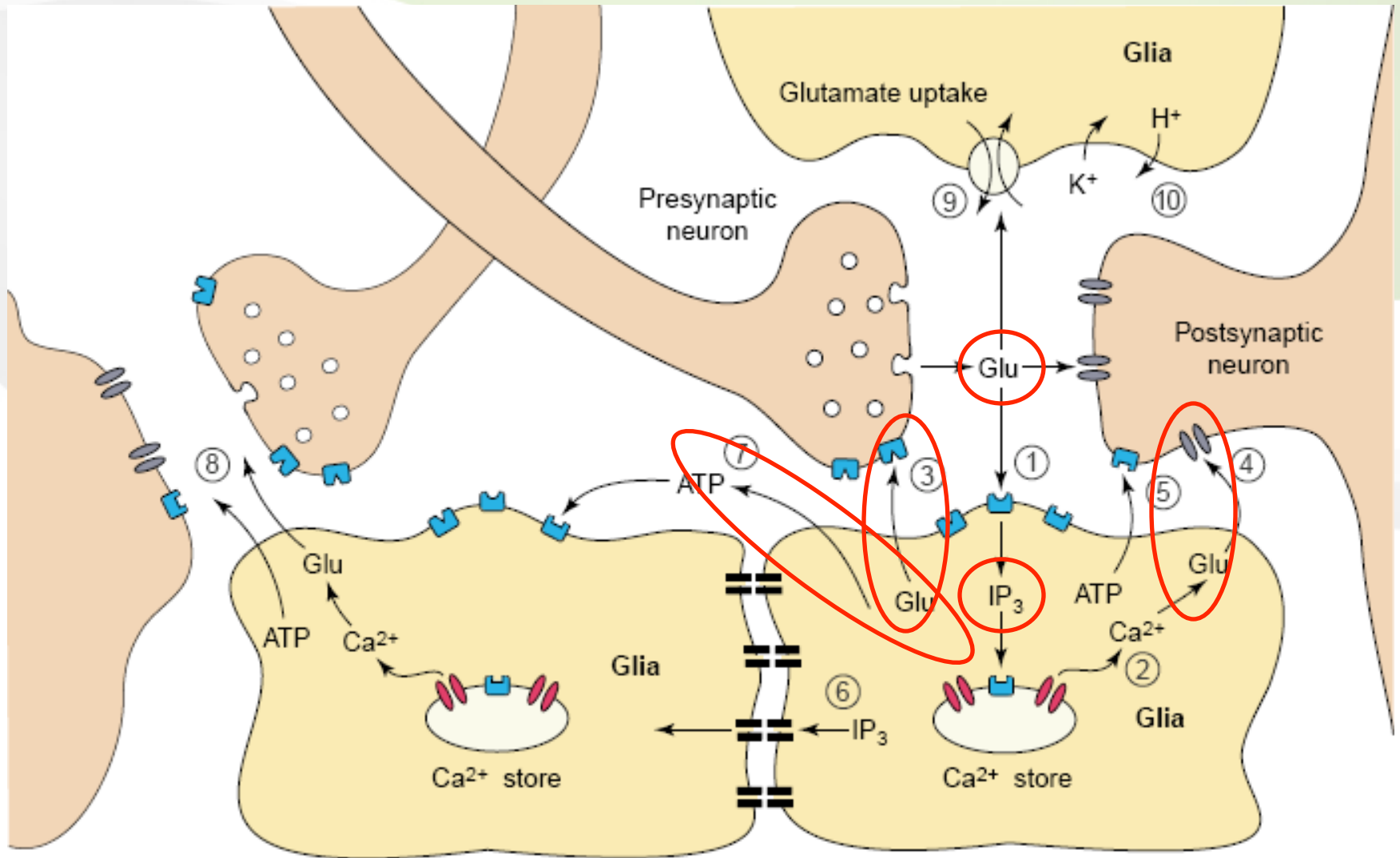


IP<sub>3</sub> receptors are then re-activated and release Ca<sup>2+</sup> in the cell cytoplasm

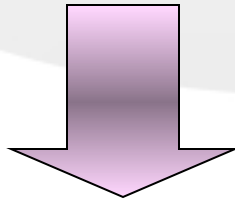


An auto-catalytic process starts

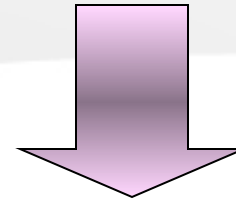
# The tripartite synapse



Over a certain threshold,  $[Ca^{2+}]$  in the cell cytoplasm activates pumps bringing  $Ca^{2+}$  in the endoplasmic reticulum and outside cells.

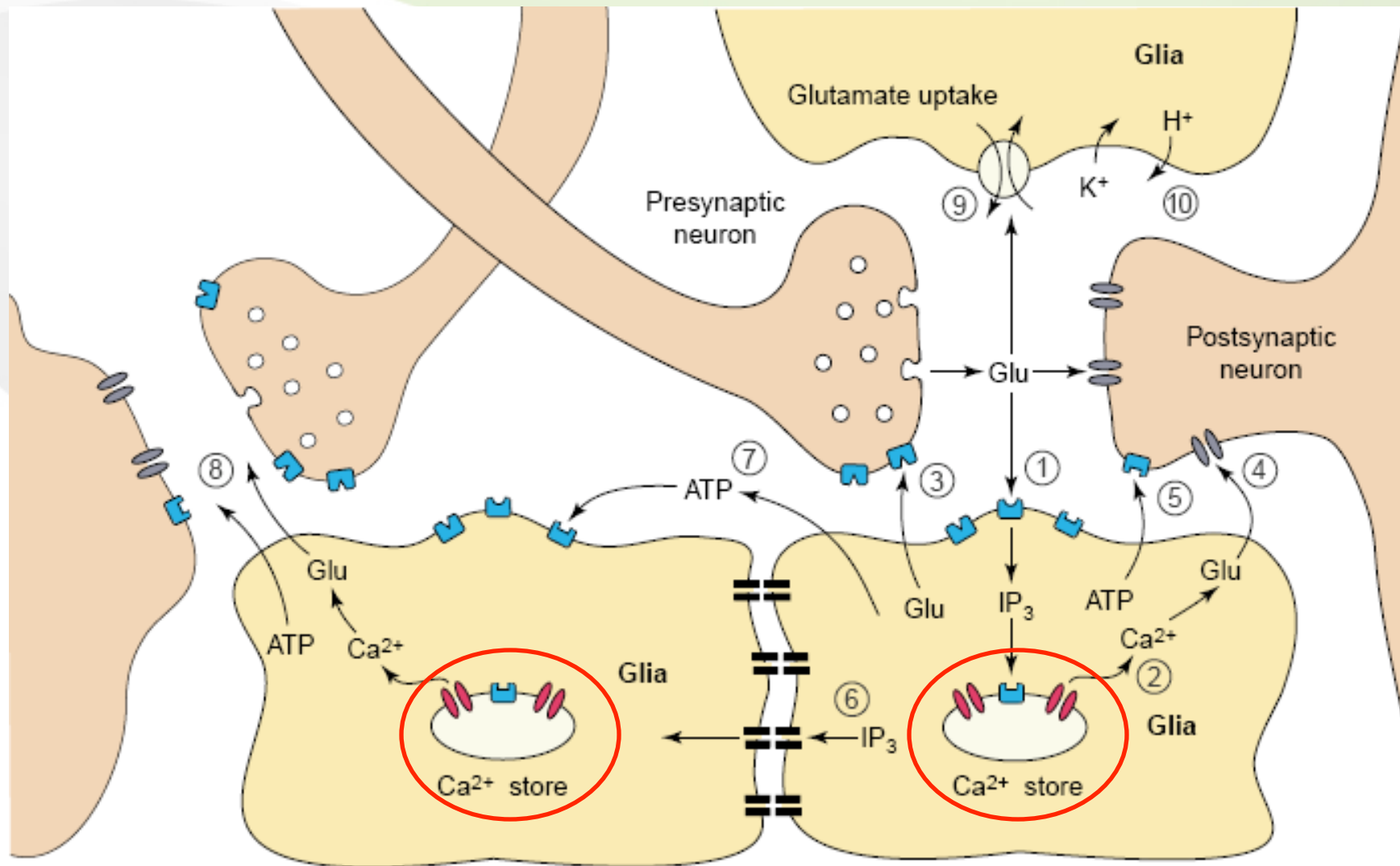


INTERcellular waves

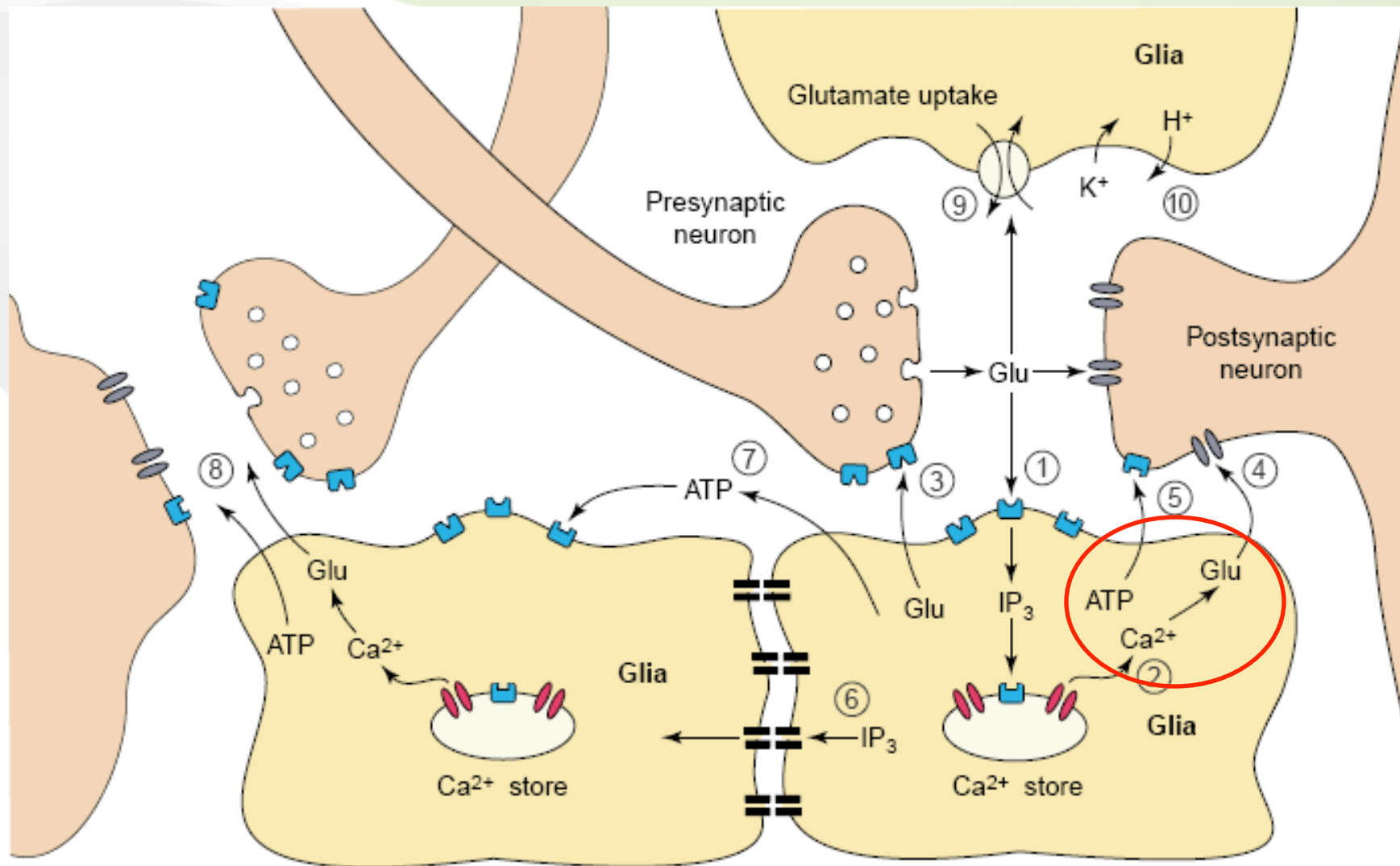


INTRAcellular waves

# The tripartite synapse



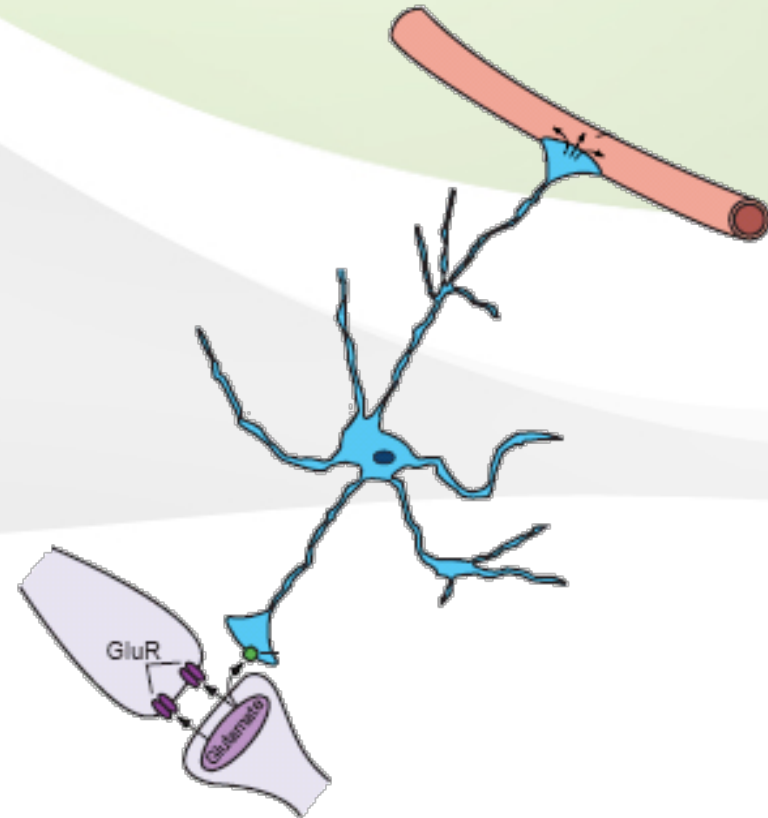
# The tripartite synapse



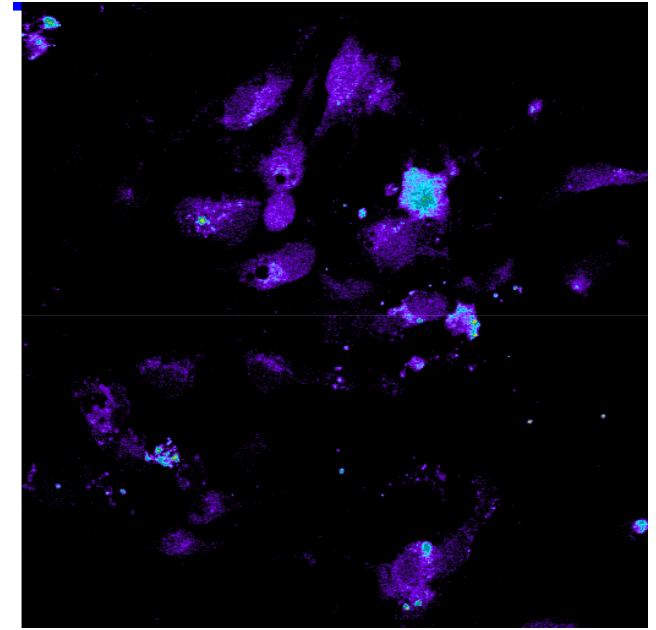
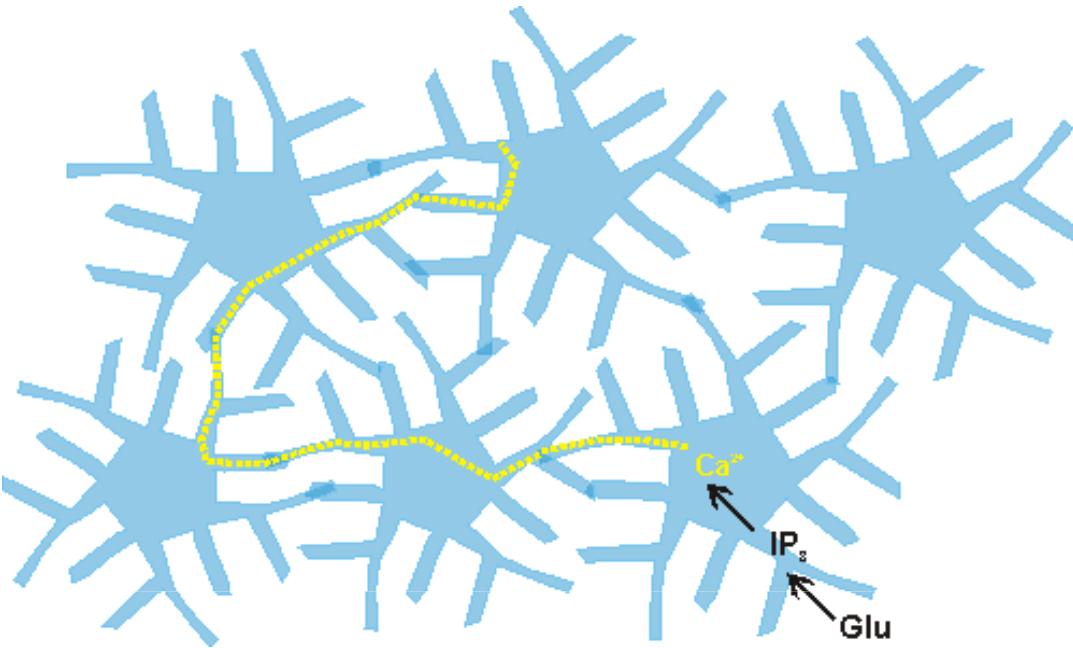
# Neuron-to-astrocyte signaling is central to the dynamic control of brain microcirculation

Zonta et al., 2003

- ↑ Neural Activity
- ↑  $\text{Ca}^{++}$  propagation throughout astrocytic syncytium
- ↑  $[\text{Ca}^{++}]$  at endfeet attached to endothelial cells
- ↑ Vesicular release of prostanoids
- ↑ Relaxation of capillary walls; decrease in vascular tone
- ↑ **Bloodflow**



## Astrocytes and calcium

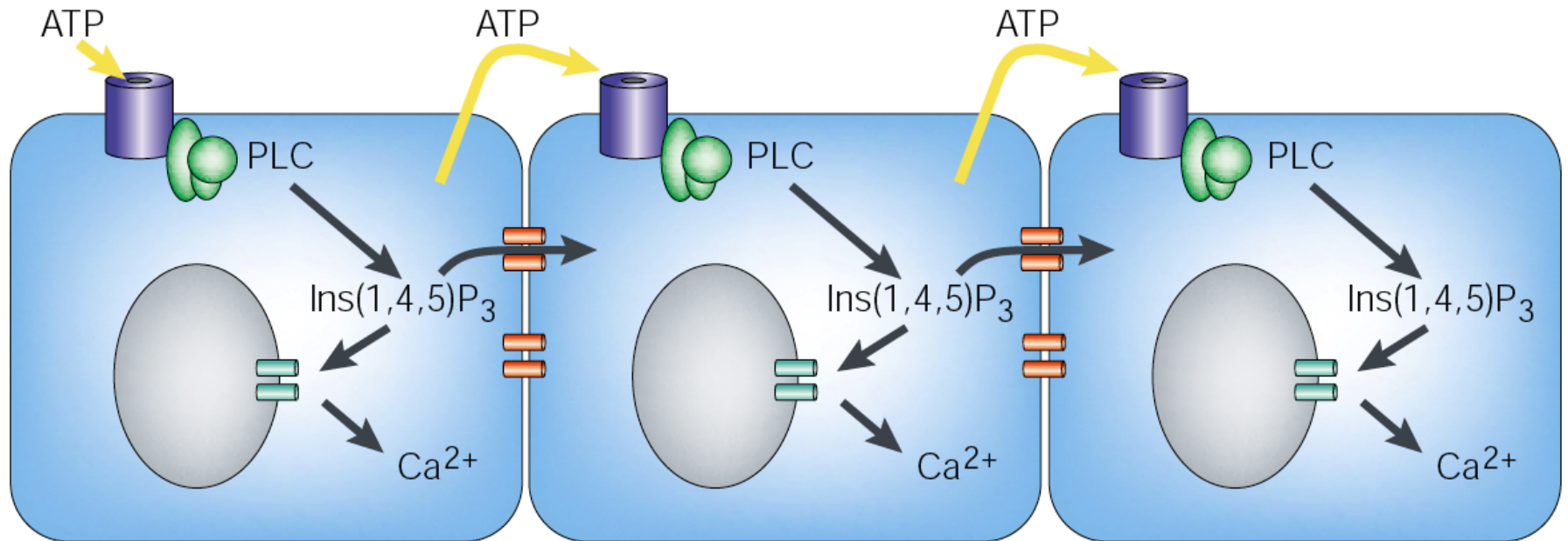


- Calcium waves propagate through the syncytium **GAP JUNCTIONS**, a non-synaptic means of communication within the brain
- Waves can be induced by mechanical stimulation and by glutamate
- Influx of calcium leads to calcium-sensitive release and uptake of ions and neuromodulators



# Ca<sup>2+</sup> Waves

(Cornell-Bell et al., M. Sanderson, A. Charles)

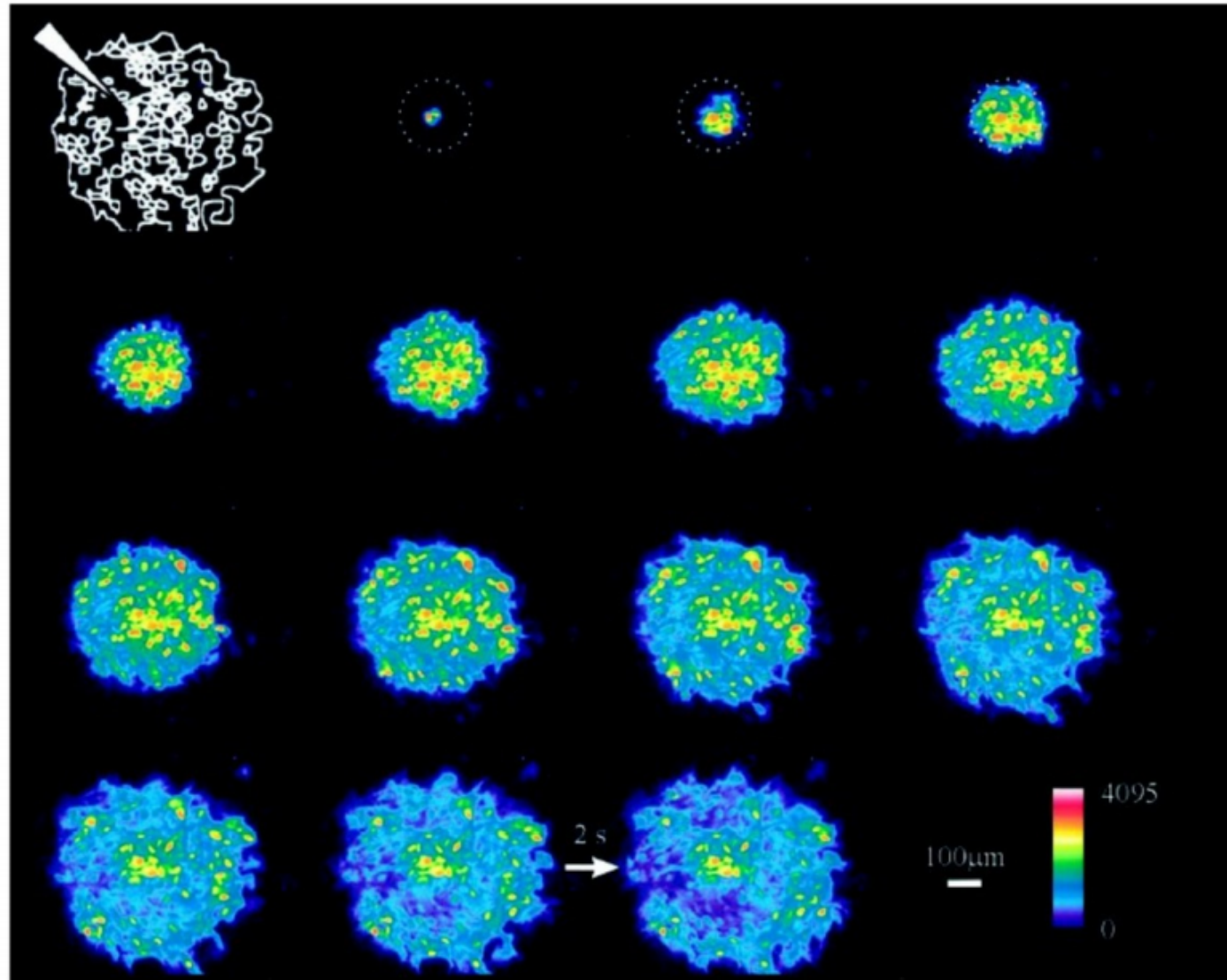


Speed: ~20 $\mu$ m/s

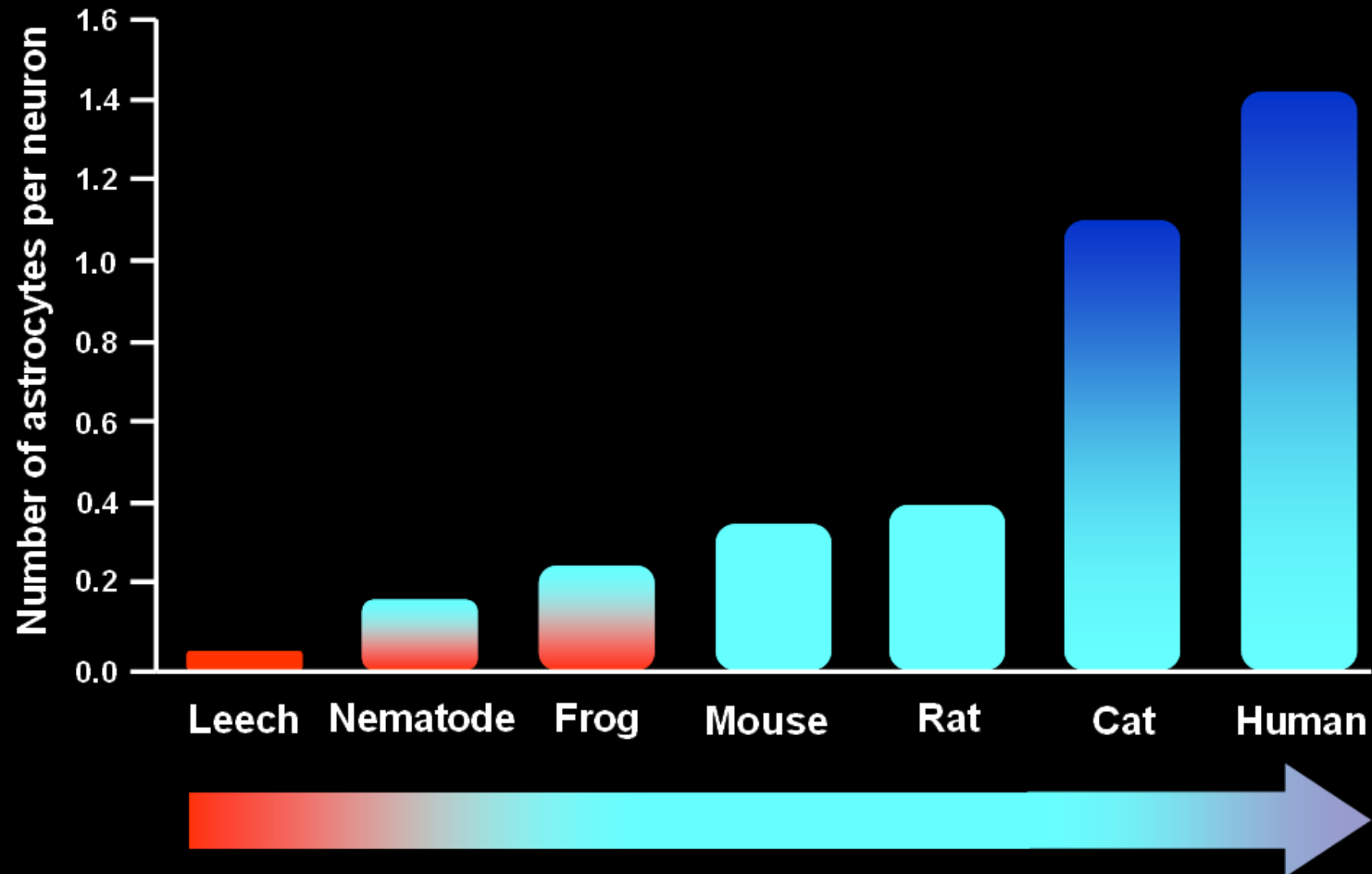
Range: a few hundred  $\mu$ m

Time scale: seconds to minutes

Ca<sup>2+</sup> waves  
have been  
observed  
in the  
hippocampus



# Relative ratios of astrocytes to neurons



Bass et al., 1971  
Sulston et al., 1983  
Nedergaard et al., 2003; modified

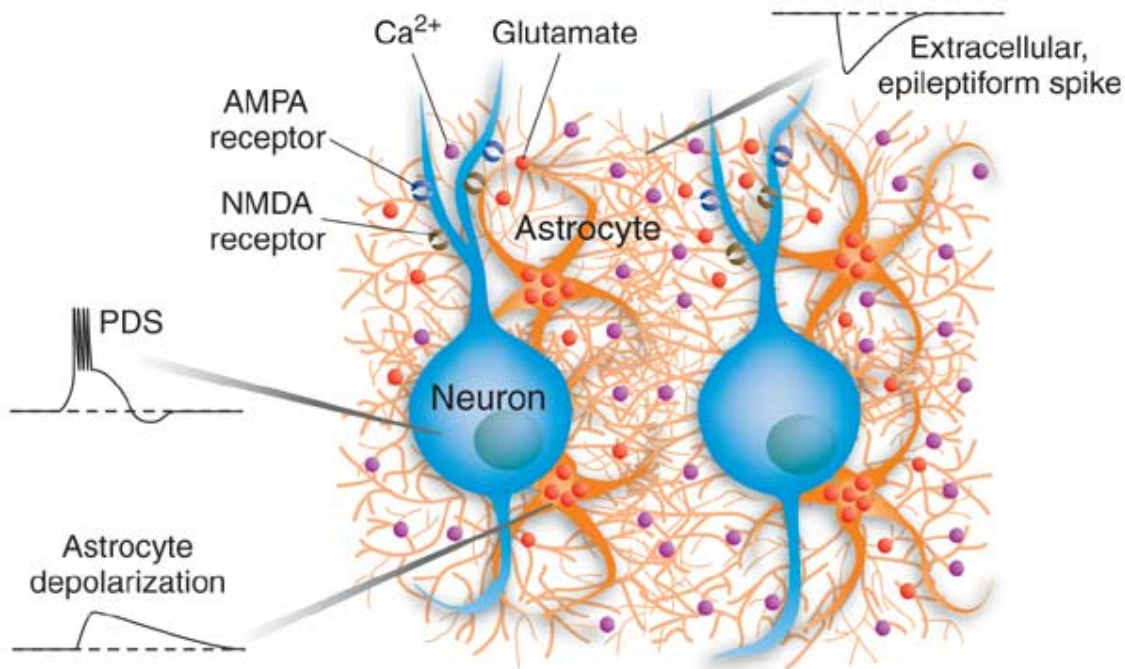
**Brain complexity and size**

A single astrocyte can cover 20 000 – 100 000 synapses in rodents...  
and possibly up to 2 million in primates and humans.

# Astrocytes and Epileptic Seizures

Tian et al.: *An astrocytic basis of epilepsy* (Nature Medicine, 11 (2005))

Epileptic discharges through local paroxysmal depolarization shift (PDS) driving groups of neurons into synchronous bursting activity.



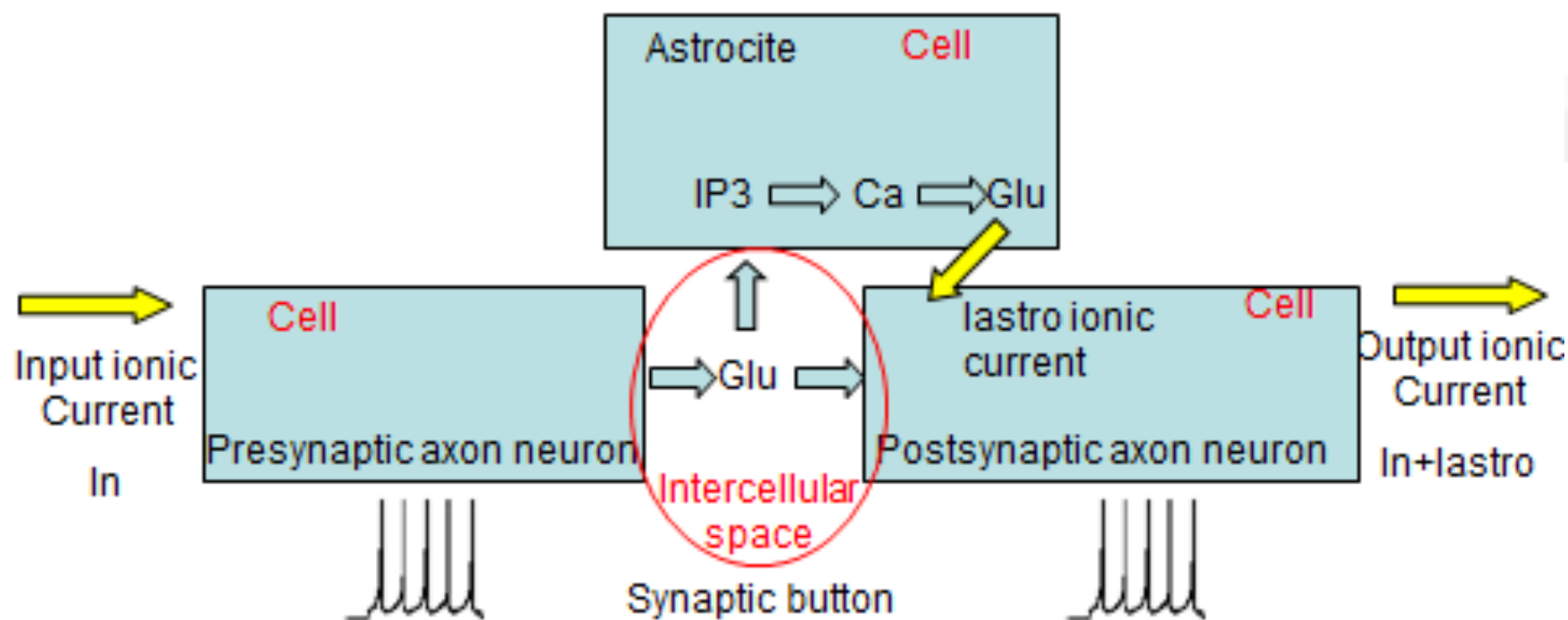
--  $\text{Ca}^{2+}$  increased in Astrocyte

– PDS - like epileptiform responses in neighboring neurons

– PDS in nearby neurons in in-vitro epilepsy models with blocked synaptic transmission

– Anti-epileptics reduced  $\text{Ca}^{2+}$  signal in astrocyte

## Biological Model of tripartite synapse

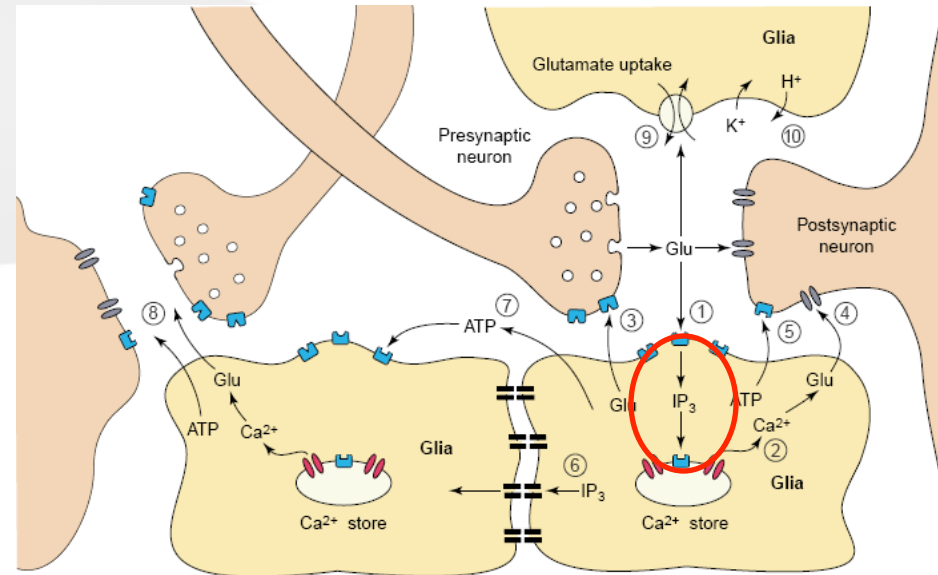


# Modelling neuron-astrocyte interactions

The intracellular IP<sub>3</sub> production can be modelled by:

$$\frac{d[IP_3]}{dt} = \frac{1}{\tau_{IP_3}} ([IP_3]^* - [IP_3]) + r_{IP_3} \Theta(v - 50 \text{ mV})$$

where [IP<sub>3</sub>]<sup>\*</sup> is the equilibrium concentration.  $\tau$  is the IP<sub>3</sub> degradation time constant and  $r$  is the production rate of IP<sub>3</sub> in response to an action potential

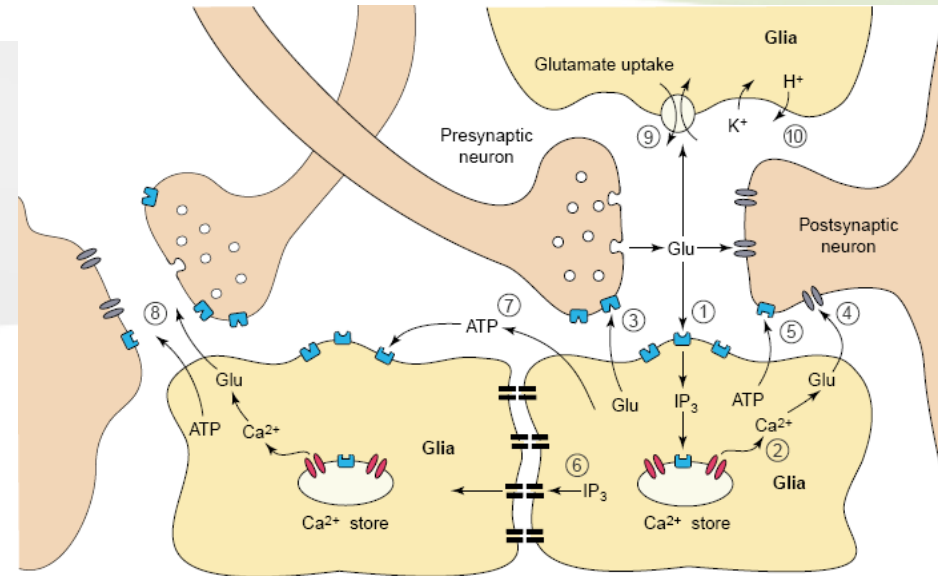


# Modelling astrocyte-astrocyte interactions

The flux of IP<sub>3</sub> can be modelled by:

$$J_G = \sum_{\langle j \rangle} \kappa \left( [IP_3]_j - [IP_3]_i \right)$$

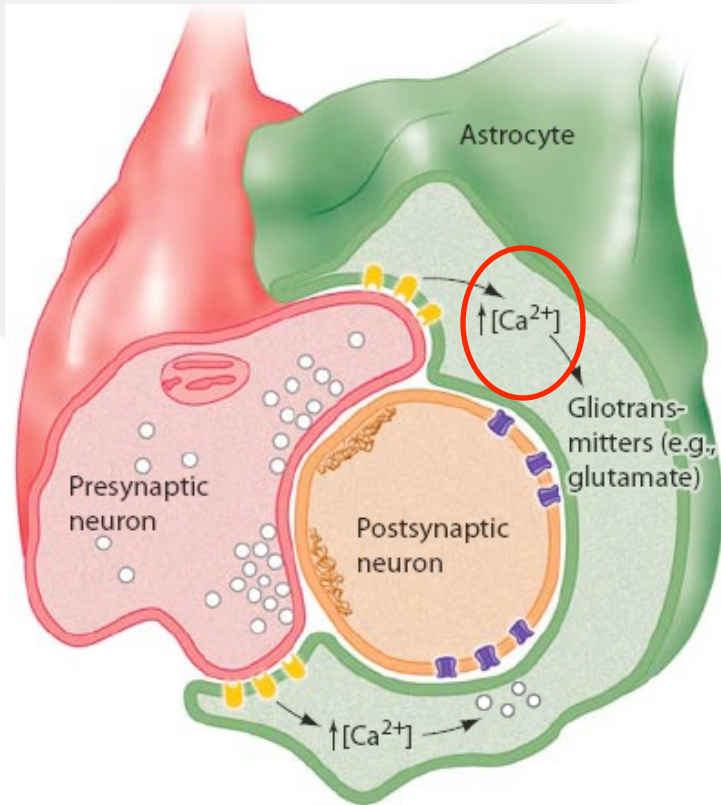
Where  $i$  indicate the  $i$ th astrocyte,  $k$  is the diffusion coupling coefficient through the gap-junction and  $\langle j \rangle$  is the contribution of the neighbouring astrocytes





Robb-Gaspers L.D. and Thomas A.P. Coordination of calcium signaling by intercellular propagation of calcium waves in the intact liver. J. Biol. Chem., 270, 8102-8107, 1995.

Ullah G., Jung P. and Cornell-Bell A.H. Anti-phase calcium oscillations in astrocytes via inositol (1, 4, 5)-triphosphate regeneration. Cell Calcium, 39, 197-208, 2006

# The Li-Rinzel model of Astrocyte



-  Extrasynaptic, NR2B-containing, NMDA receptors
-  Metabotropic glutamate receptors

$$\frac{d[Ca^{2+}]}{dt} = -J_{channel}(q) - J_{pump} - J_{leak}$$

$$\frac{dq}{dt} = \alpha_q(1 - q) - \beta_q q$$

$$J_{channel} = c_1 v_1 m_\infty^3 n_\infty^3 q^3 ([Ca^{2+}] - [Ca^{2+}]_{ER})$$

$$J_{pump} = \frac{v_3 [Ca^{2+}]^2}{k_3^2 + [Ca^{2+}]^2}$$

$$J_{leak} = c_1 v_2 ([Ca^{2+}] - [Ca^{2+}]_{ER})$$

$$m_\infty = \frac{[IP_3]}{[IP_3] + d_1}$$

$$n_\infty = \frac{[Ca^{2+}]}{[Ca^{2+}] + d_5}$$

$$\alpha_q = a_2 d_2 \frac{[IP_3] + d_1}{[IP_3] + d_3}$$

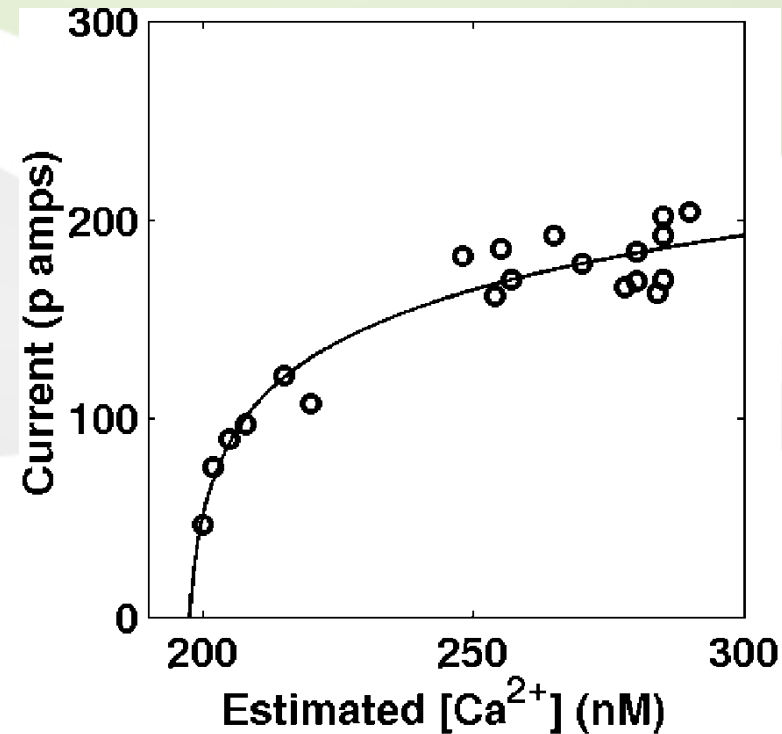
$$\beta_q = a_2 [Ca^{2+}]$$



# Experimental model for astrocyte-neuron interaction

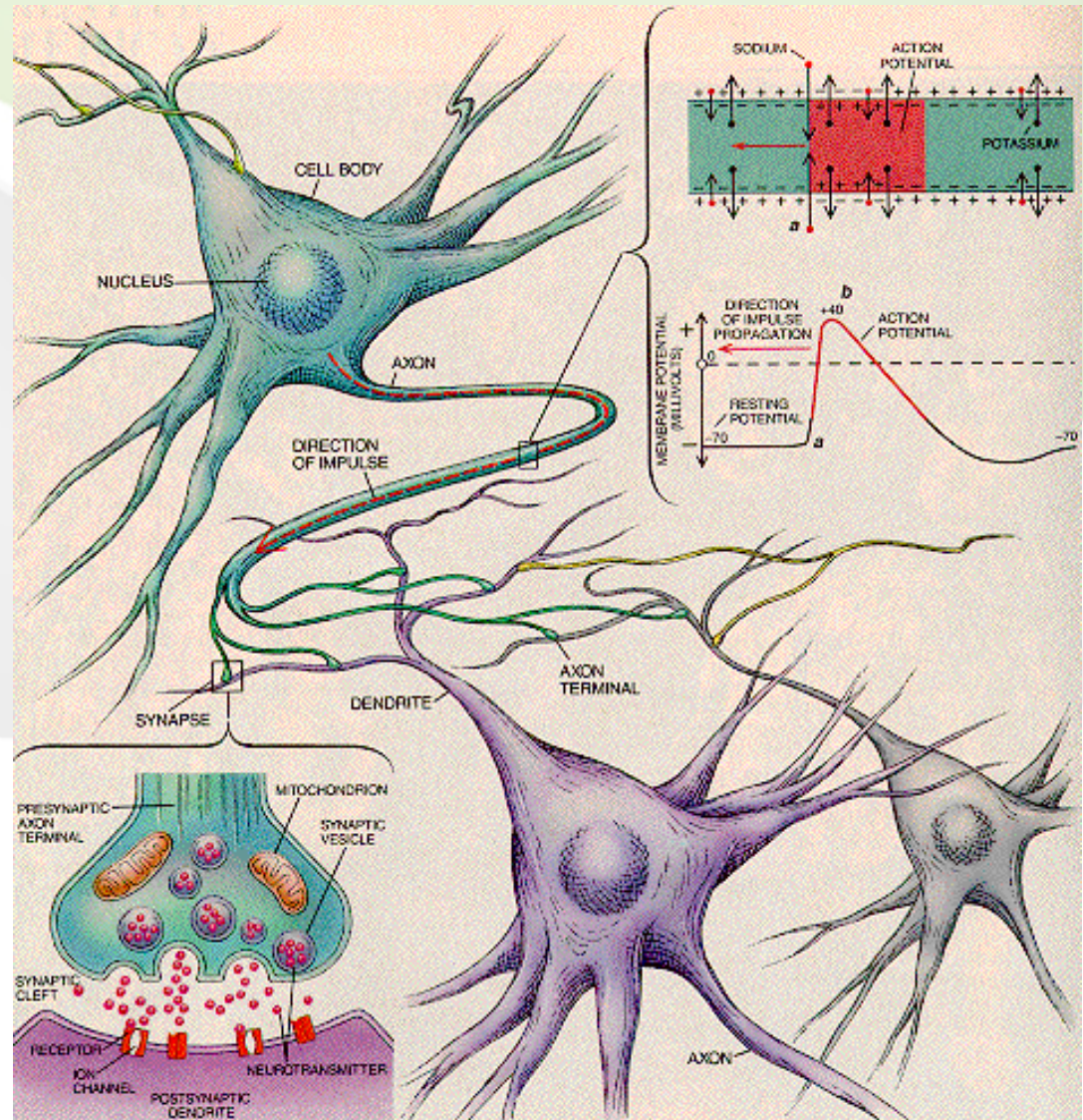
Experimental data can be useful to model the correlation of the  $Ca^{2+}$  concentration into the astrocyte environment with the weak additional synaptic currents coming from the neighbouring astrocytes

$$I_{astro} = 2.11\Theta(\ln y) \ln y$$
$$y = \frac{[Ca^{2+}]}{nM} - 196.69$$



# NEURO-ASTROCYTE MODELS

# Neuro-Astrocyte using Hodgkin Huxley

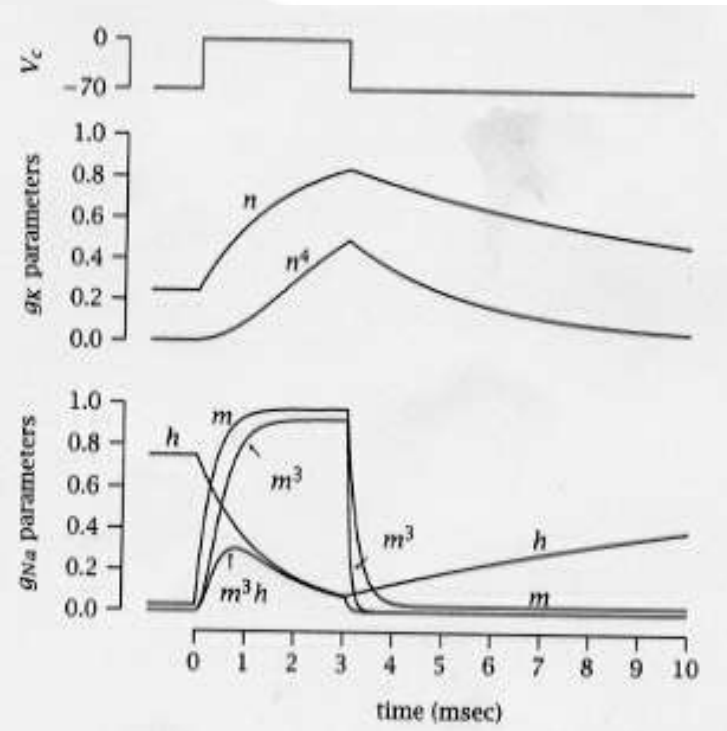


$$C_m \frac{dv}{dt} = -g_K n^4 (v - v_K) - g_{Na} m^3 h (v - v_{Na}) - g_l (v - v_l) + I_{ext} + I_{astro}$$

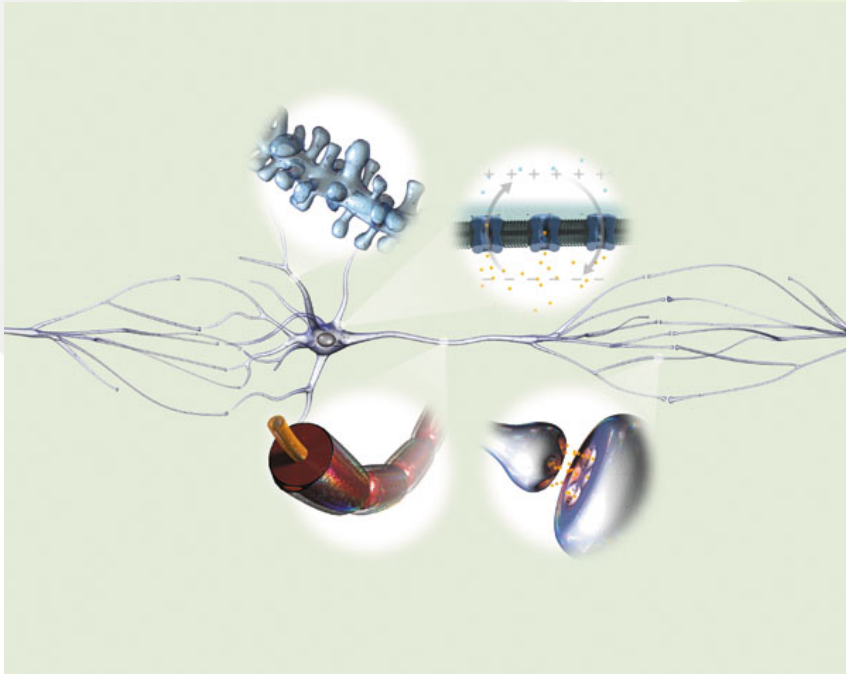
$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m$$

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h$$



# A modified Izhikevich neuronal model

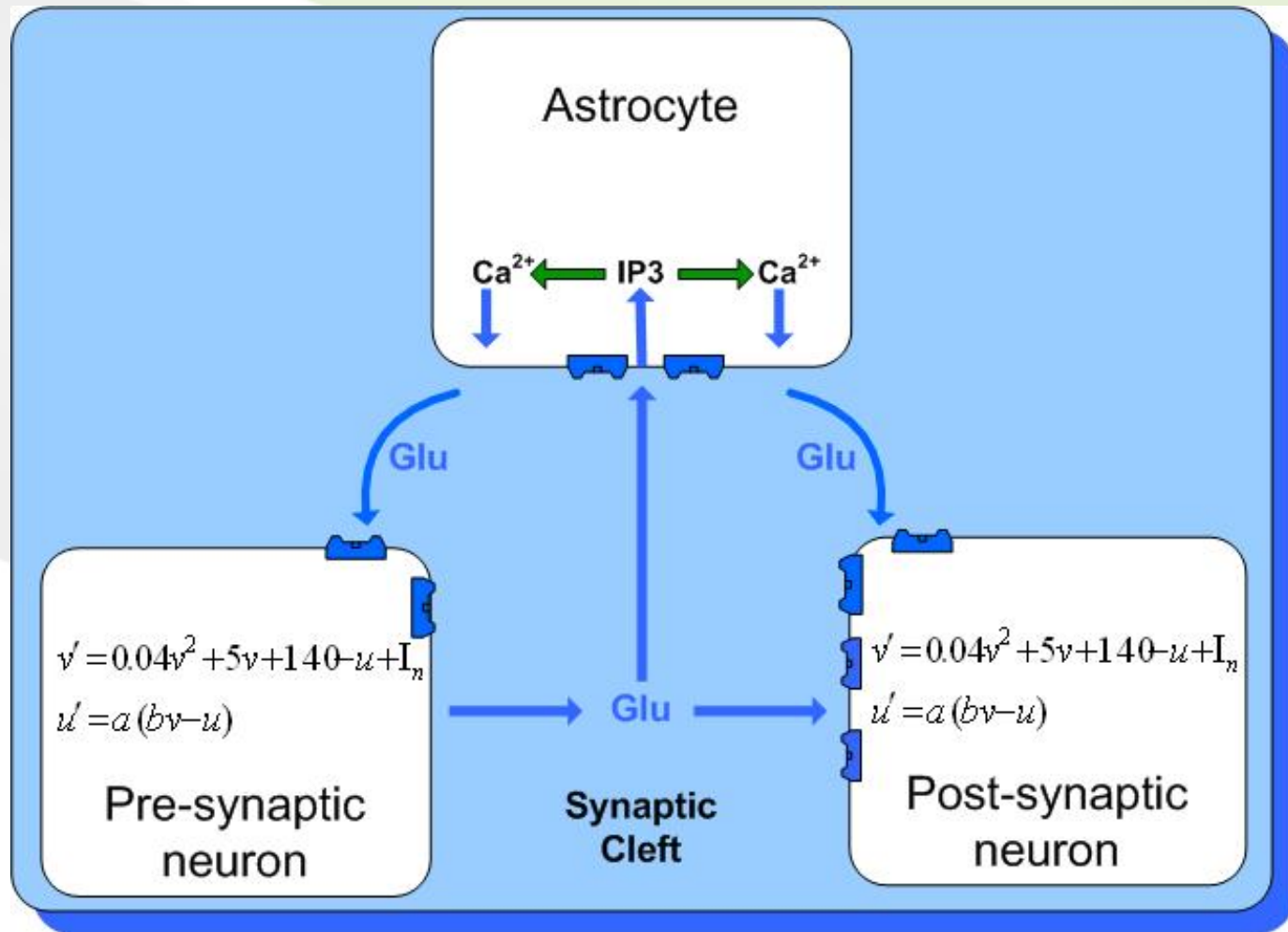


$$\text{if } v \geq +30 \text{ mV}, \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

$$v' = 0.04v^2 + 5v + 140 - u + I + I_{astro}$$

$$u' = a(bv - u)$$

# Dressed Neuron



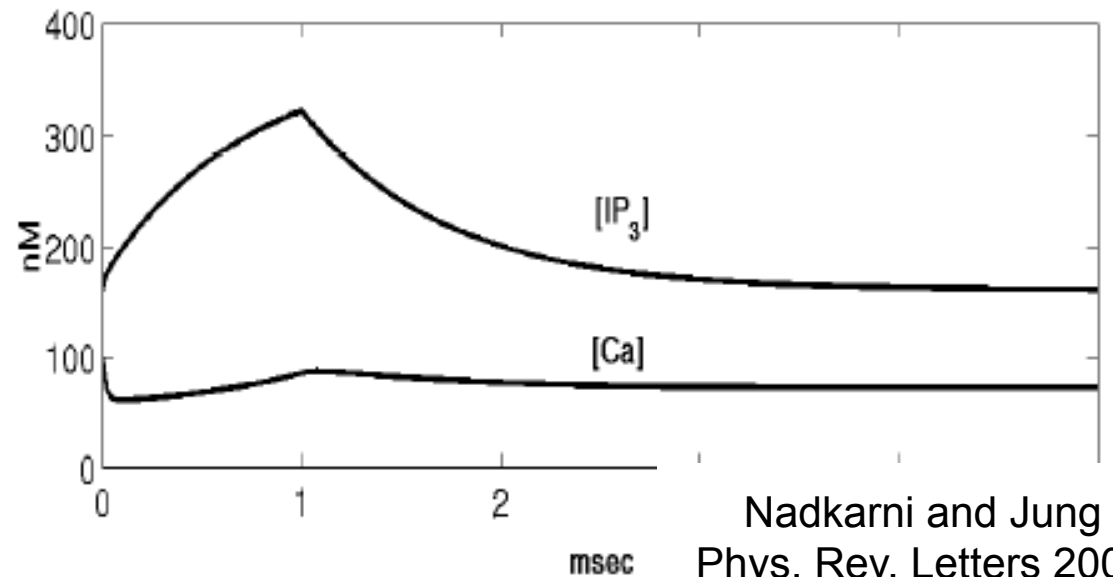
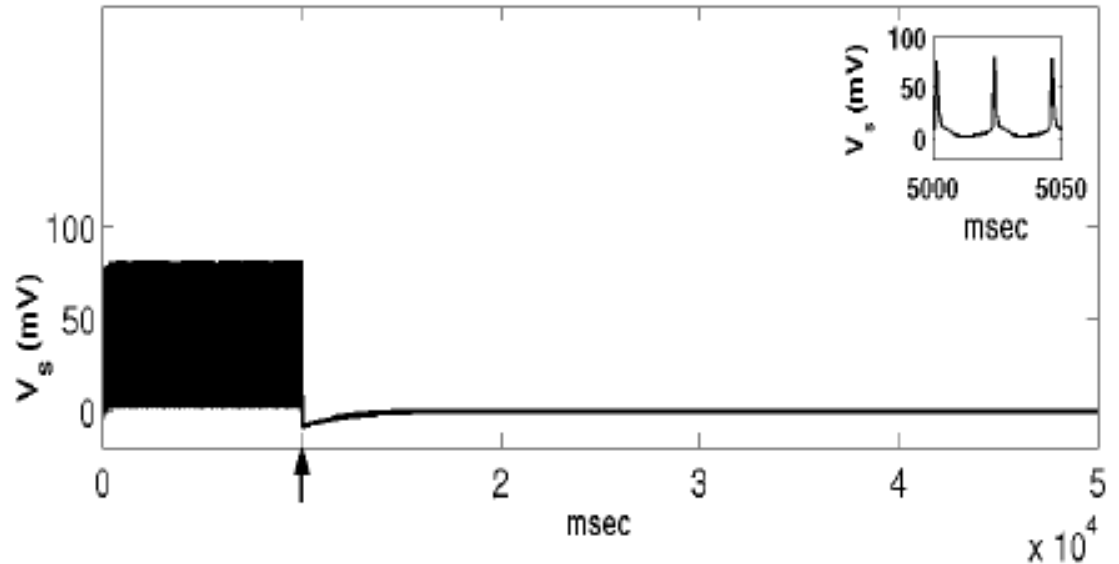
Neural Firing



[IP<sub>3</sub>] increases

[Ca<sup>2+</sup>] quite stationary

$$I_{\text{astro}} = 0$$



Nadkarni and Jung  
Phys. Rev. Letters 2003

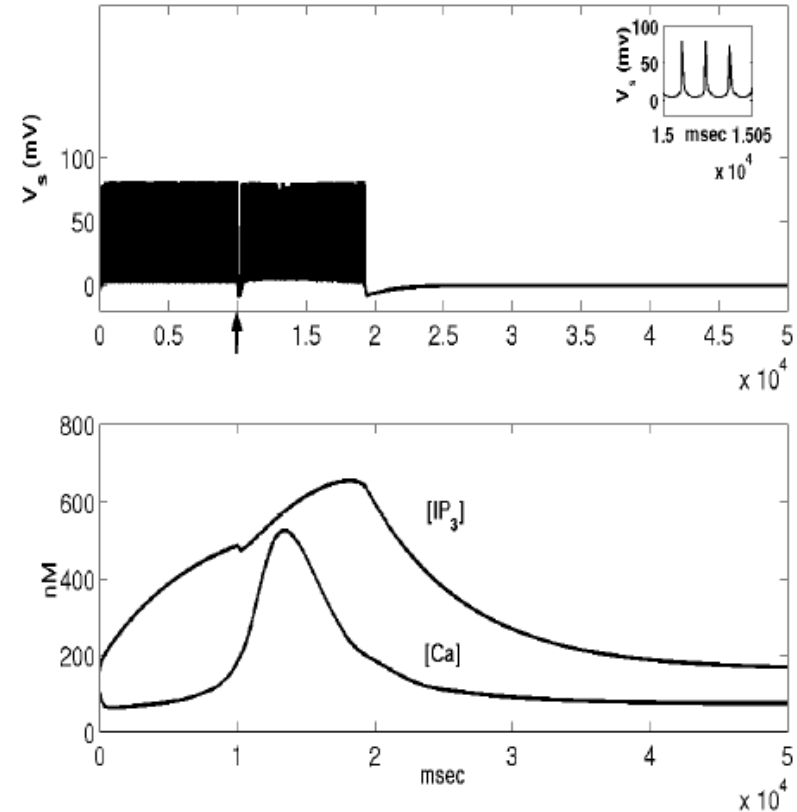
10 sec exogenous  
Current to neuron



[IP<sub>3</sub>] increases

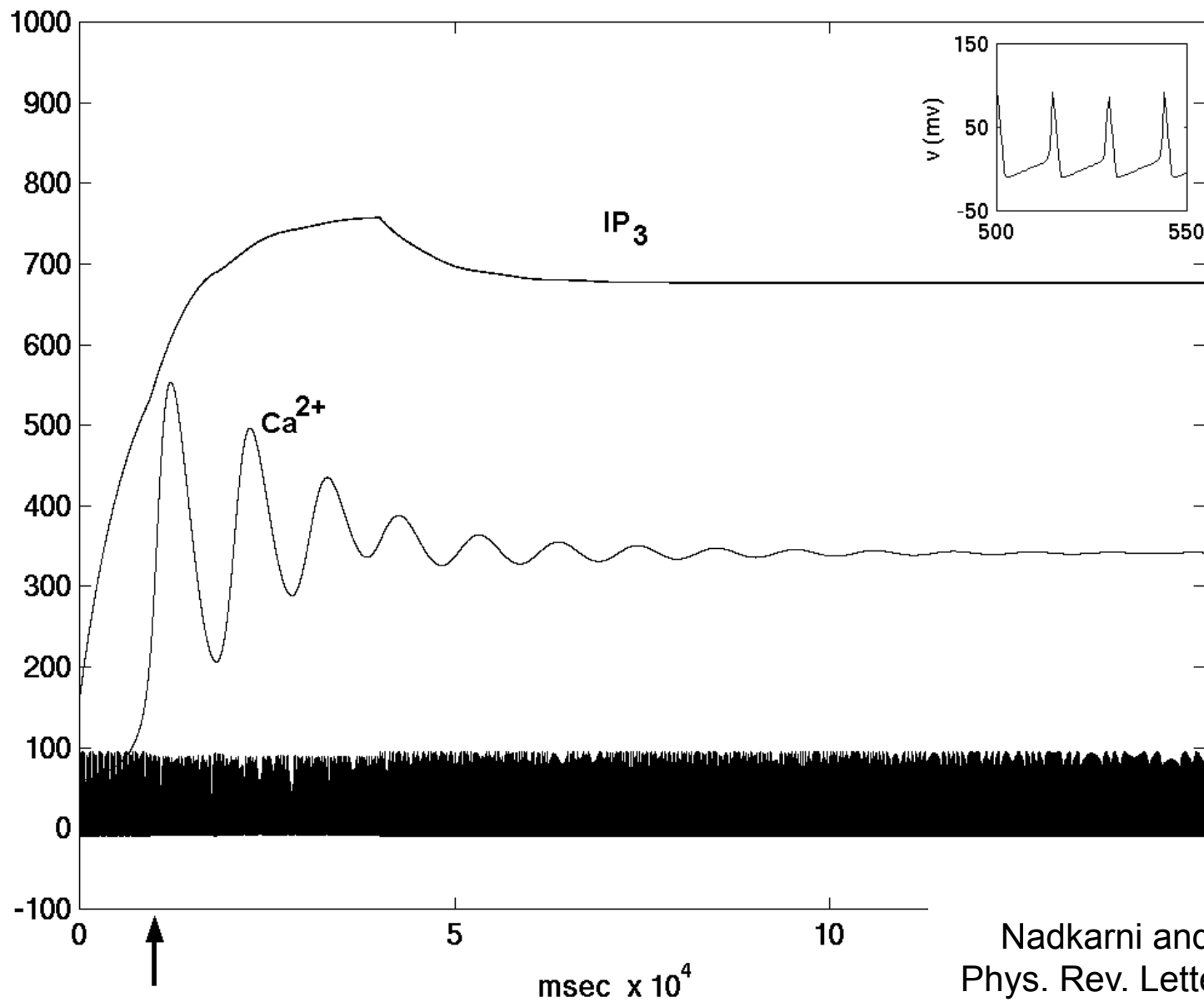


Ca<sup>2+</sup> oscillations start



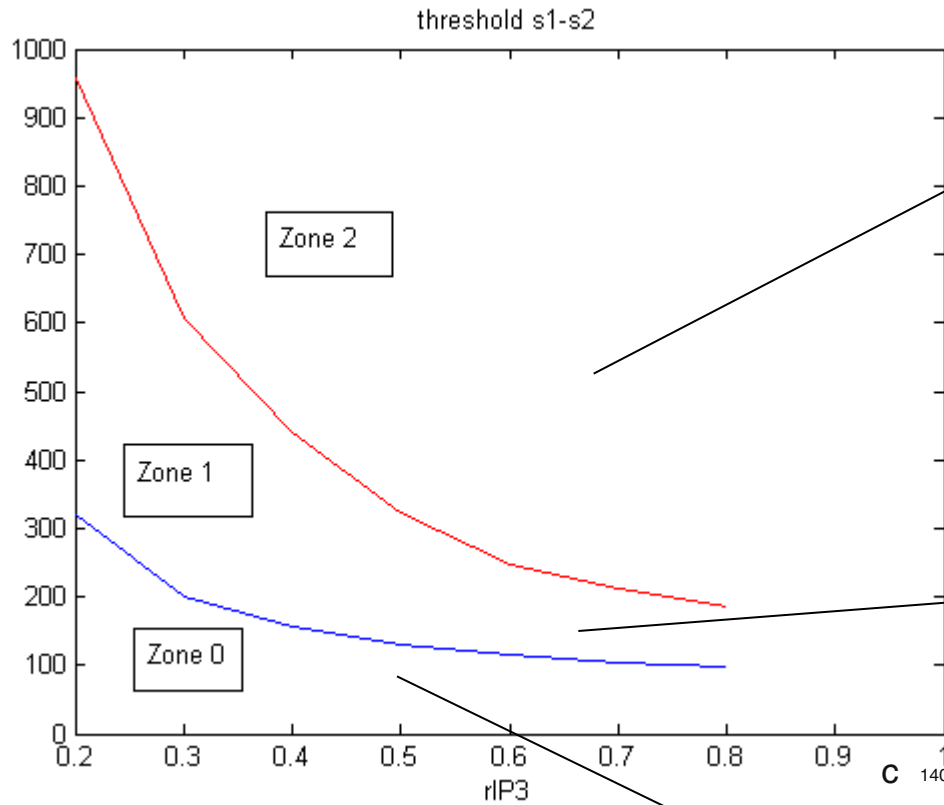
Astrocyte feedback self-sustains Neural activity!



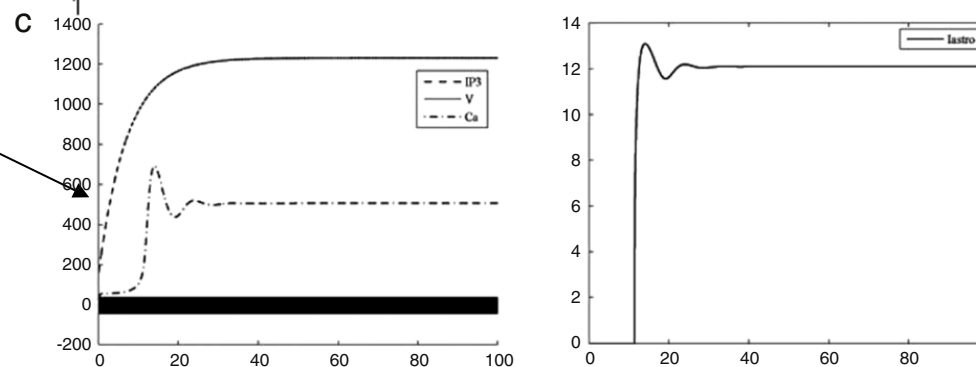
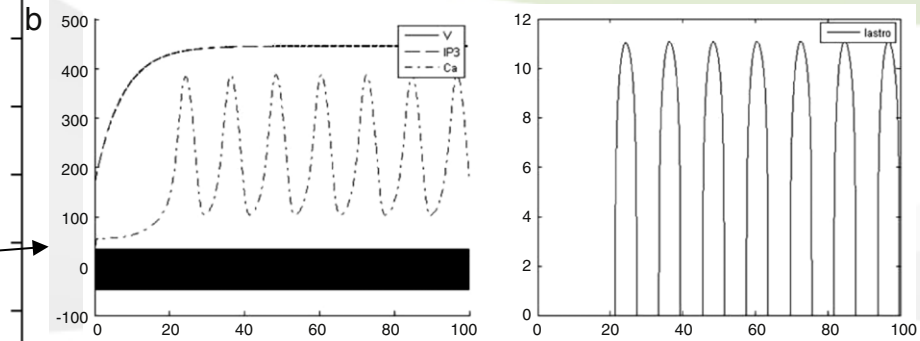
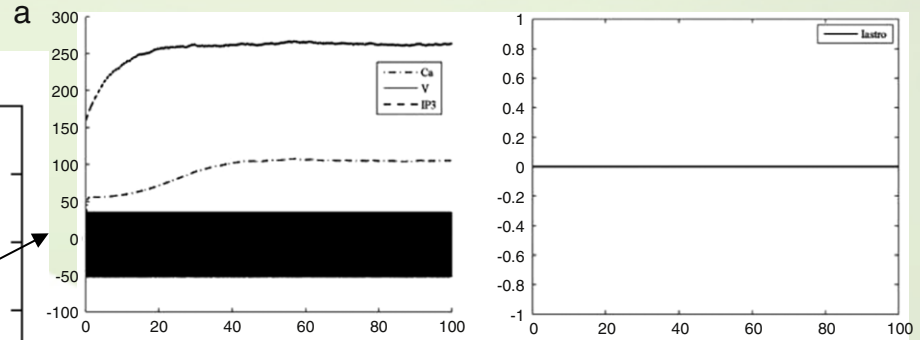


Nadkarni and Jung  
Phys. Rev. Letters 2003

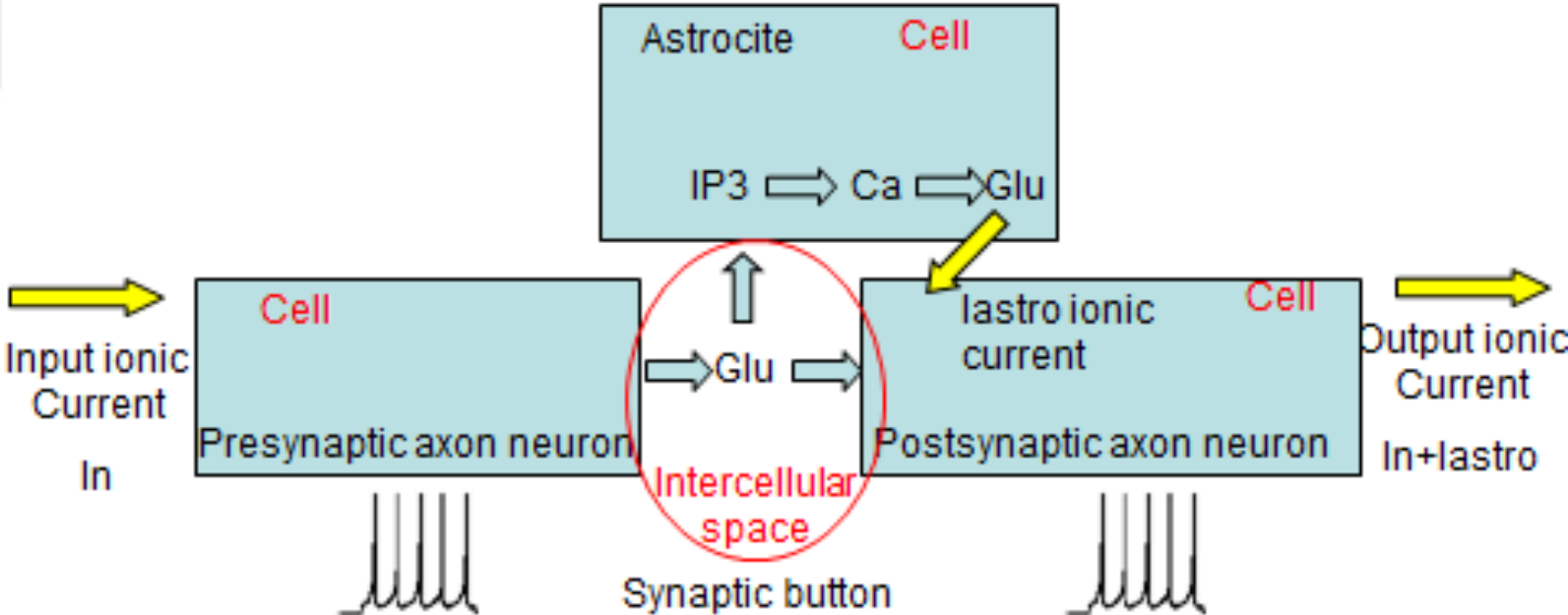
# Neuron-Astrocyte interaction



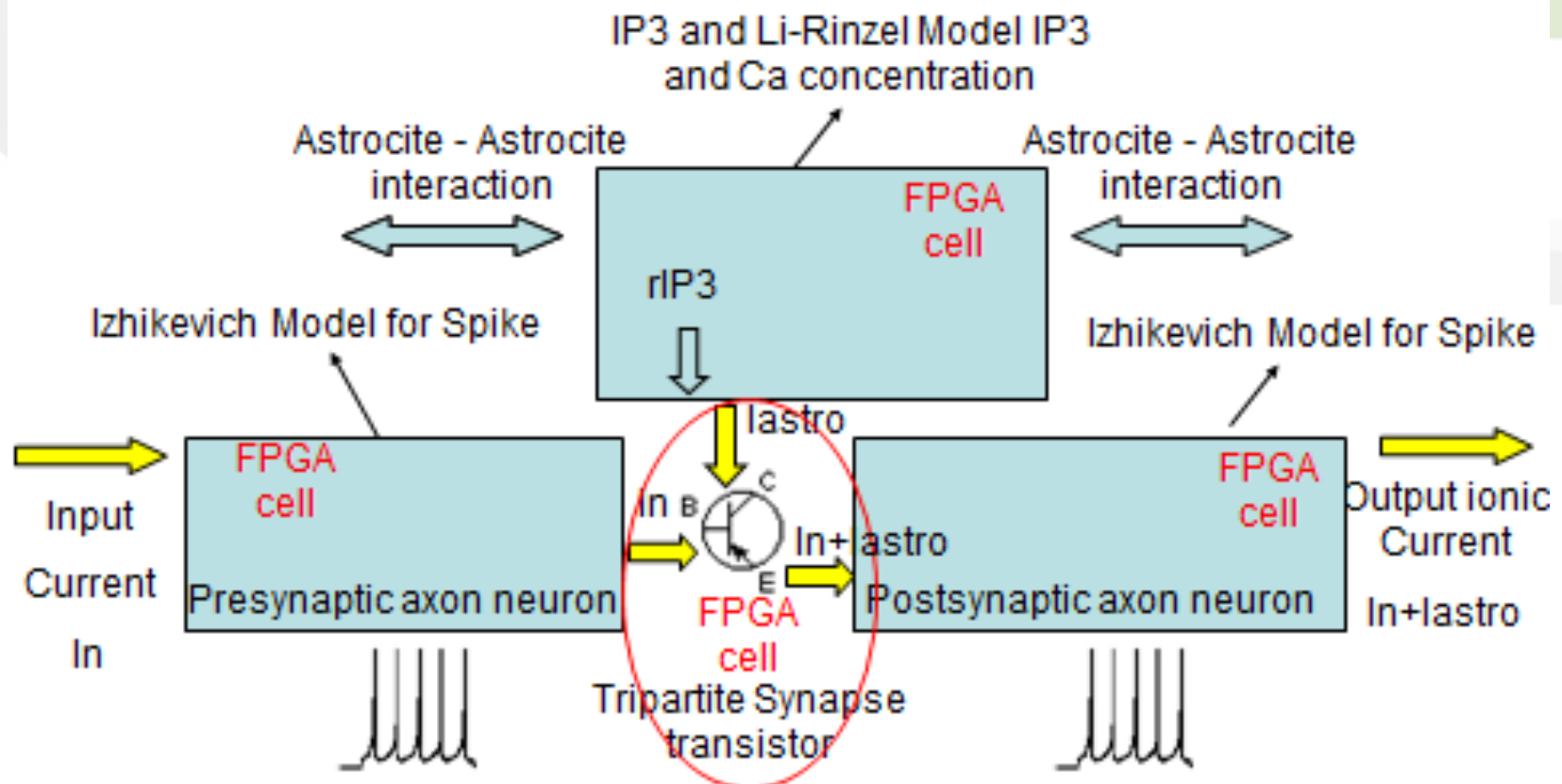
variables {  $r_{IP3}$   
 $I_{neuron}$



# Biological Model of tripartite synapse

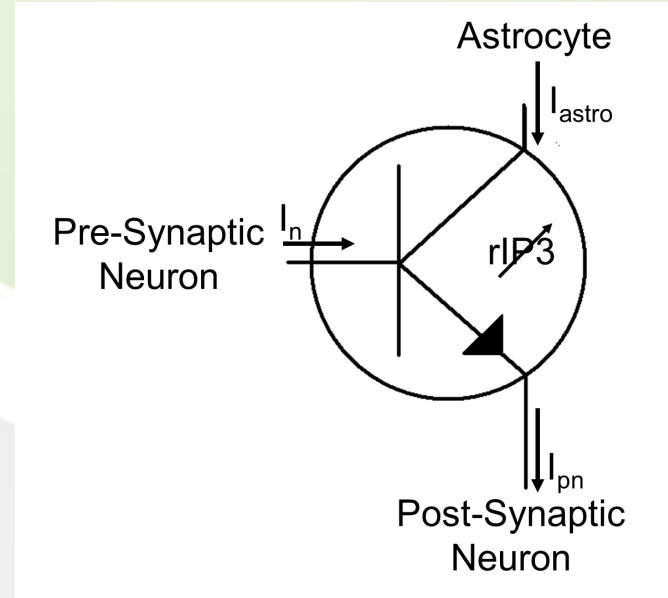


# Computational Hardware Model of tripartite synapse



# Our model: toward a transistor-based approach

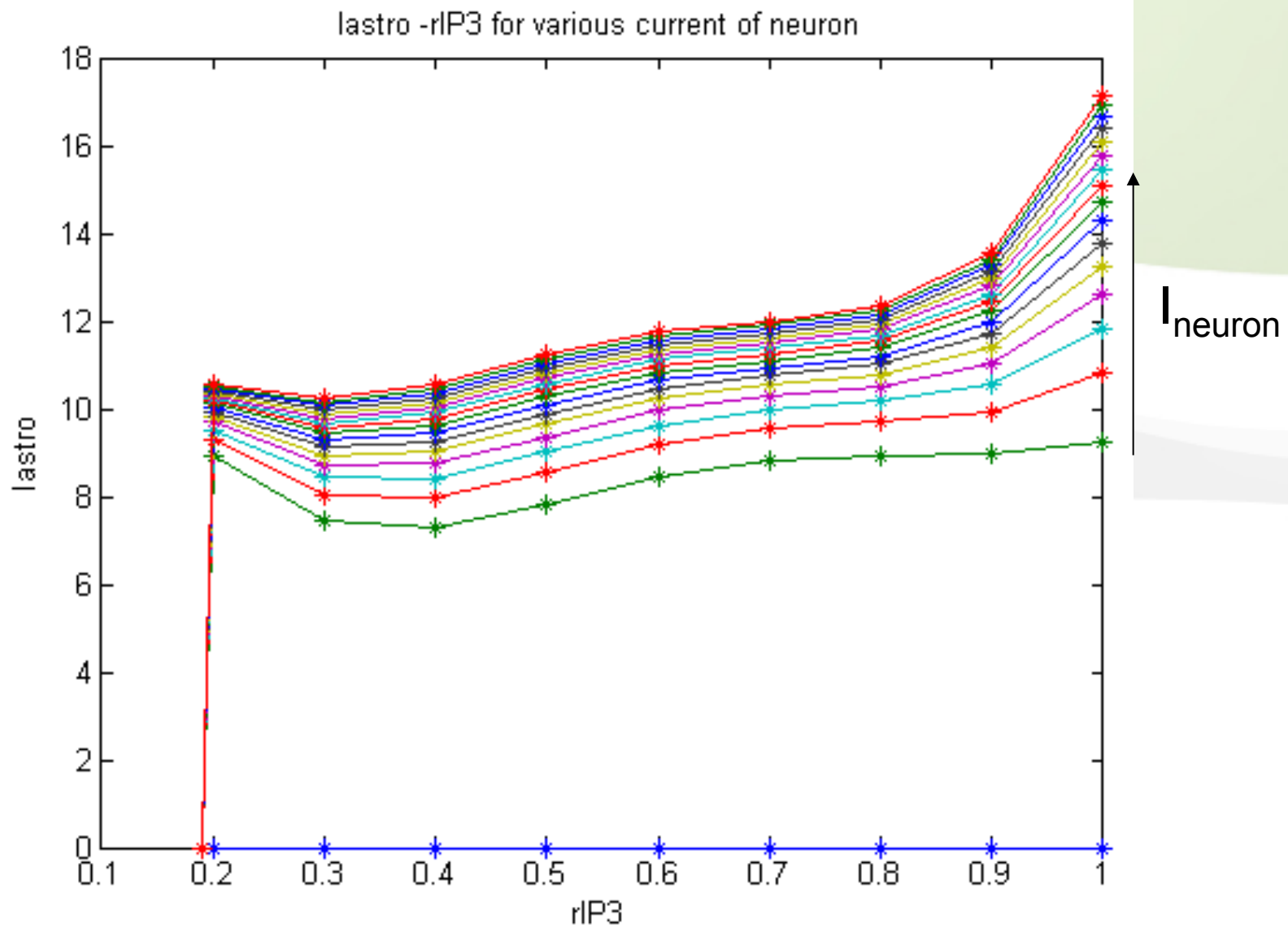
$$I_{astro} = I_{neuron} \cdot h_{fe}$$



$$h_{fe} = \left\{ \begin{array}{ll} 0 & \text{if } I_n \leq s_1(rIP3) \\ \phi(t - d_1(A_1)) A_1(rIP3, I_n, s_1) \sin(h(A_1)) & \text{if } s_1(rIP3) < I_n \leq s_2(rIP3) \\ \phi(t - d_2(I_{fin})) \left[ \frac{I_{fin} + (A_2 e^{-\frac{t}{\tau}} \sin(2\pi ft))}{I_n} \right] & \text{if } I_n > s_2(rIP3) \end{array} \right.$$

where  $s_1$  and  $s_2$  are the threshold for the zone 0,1,2

# The Neuron-Astrocyte IS a non-linear transistor



# The role of Astrocytes: Tripartite Synapses

In summary as input-output model:

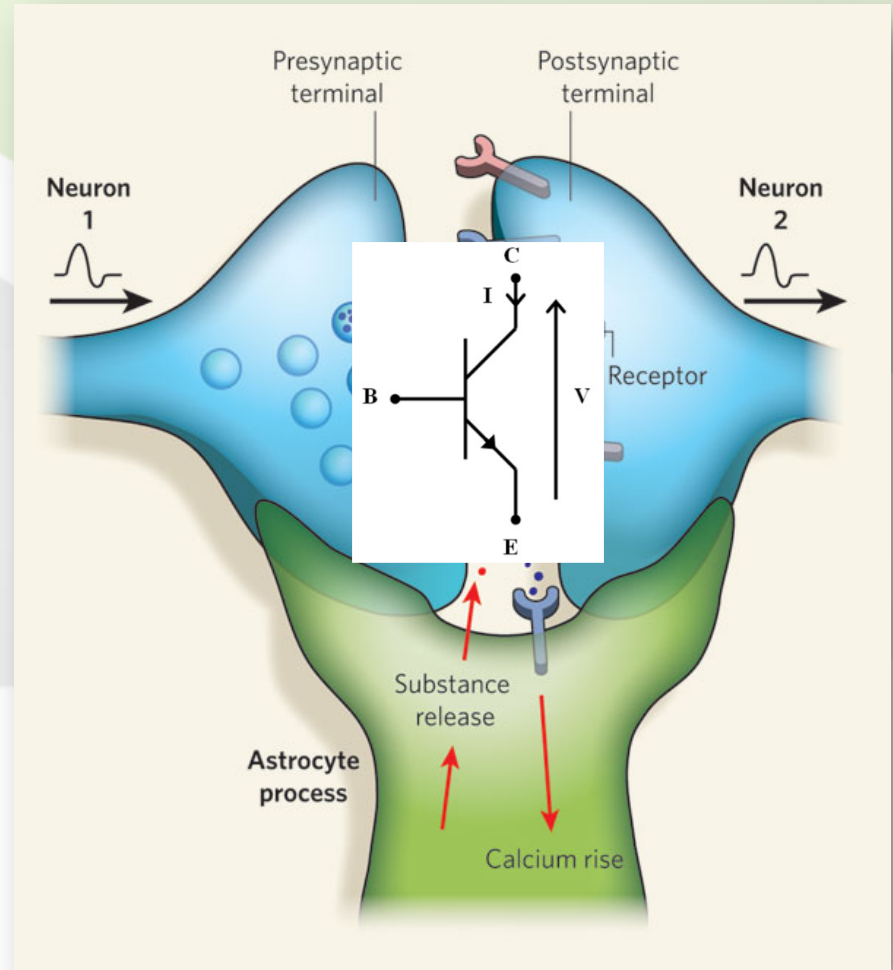
**Pre-synaptic Neuron**

↓  
**Neurotransmitters**

↓  
**Inositol 1,4,5-trisphosphate (IP3)  
rate of production (rIP3)**

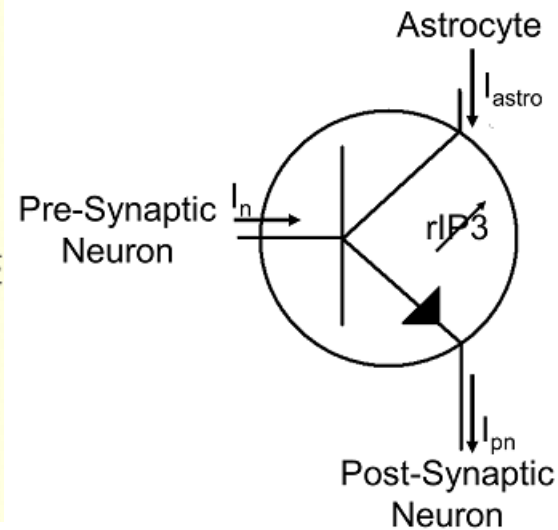
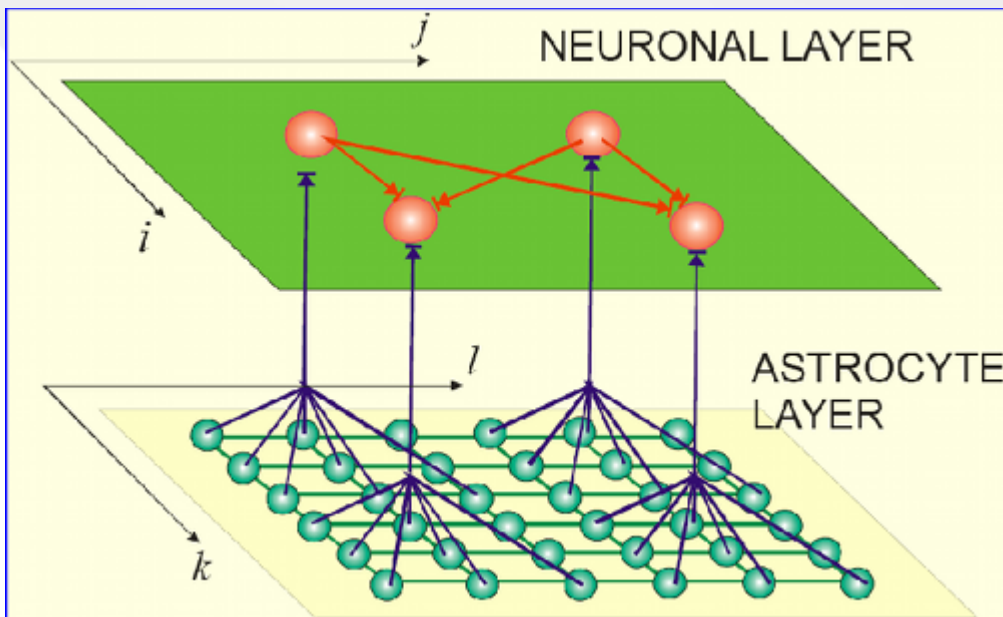
↓  
**Calcium  
Oscillation**

↓  
**Post-synaptic  
Neuron**



# Are SNAN possible?

**Develop a novel and efficient computational implementation of a Spiking Neuron-Astrocyte Network (SNAN)**





# Policronization in SNN

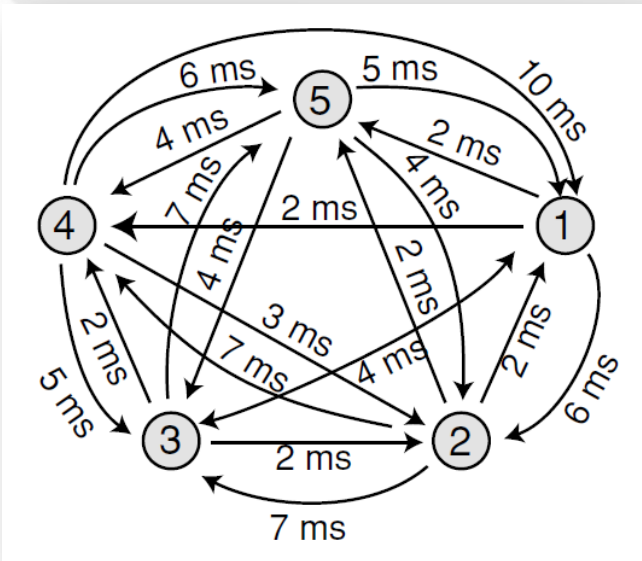
**Edelman**



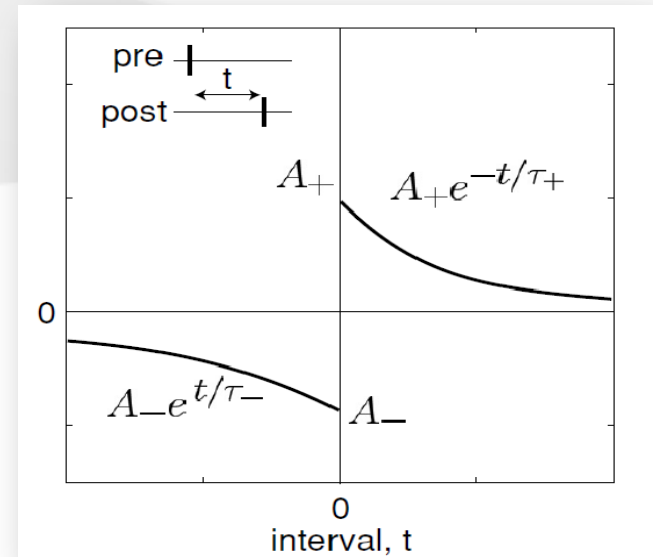
Theory of neuronal group selection  
(TNGS, Neural Darwinism)

**Izhikevich**  
**Network**

Axonal Conduction  
Delays



Spike-Timing-Dependent  
Plasticity (STDP)

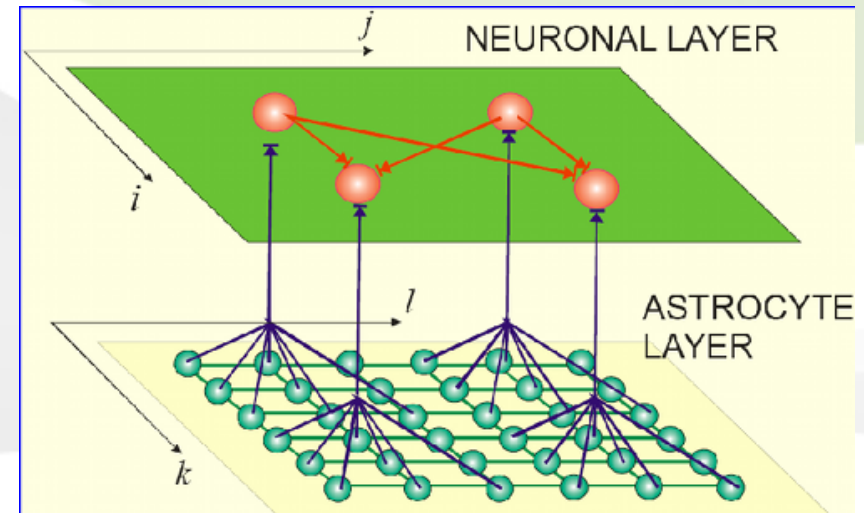
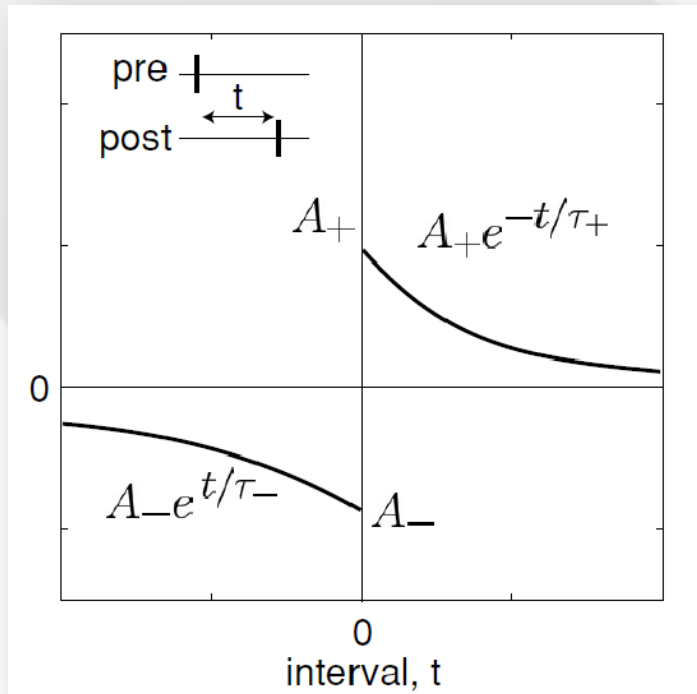


**Policronization**

# How to define a SNAN

## Two learning rules:

Neural weights are updated according to the Spike-Timing-Dependent Plasticity (STDP).



$r_{IP3}$  values are updated according to the following rule:

$$r_{IP3}(n+1) = r_{IP3}(n) + 0.05(r_{IP3}(n) - r_{IP3}(n-1))$$

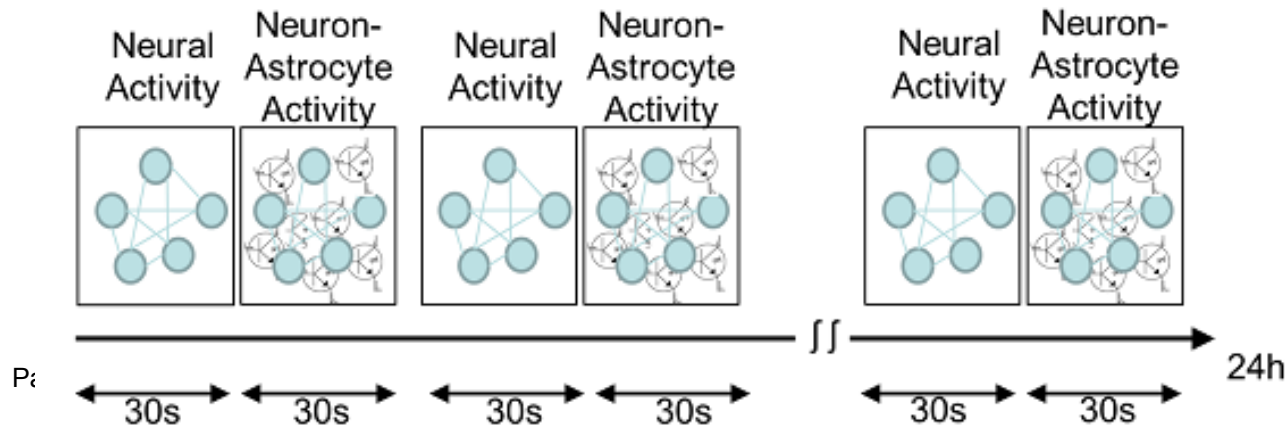
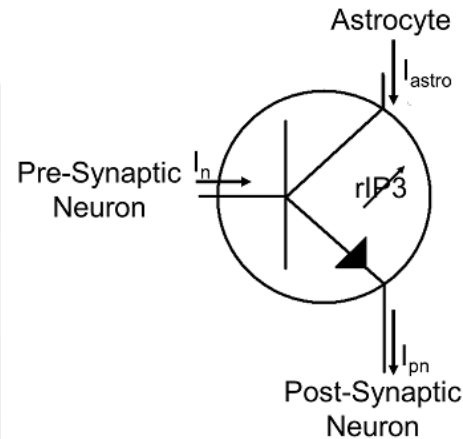
# How to define a SNAN: Timing

Inizialization of neurons

Neural activity only (30s)

Inizialization of astrocytes

Simultaneous Activity of Neurons and Astrocytes



# Experimental results

24h simulation

Evaluations after

3h

6h

12h

18h

24h

***Network dimension***

***Neurons***

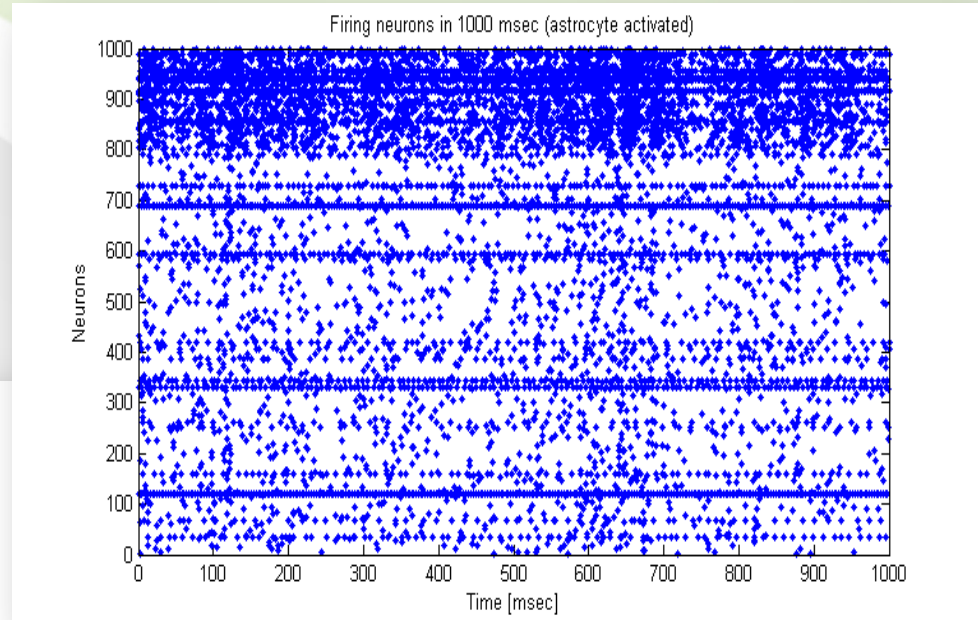
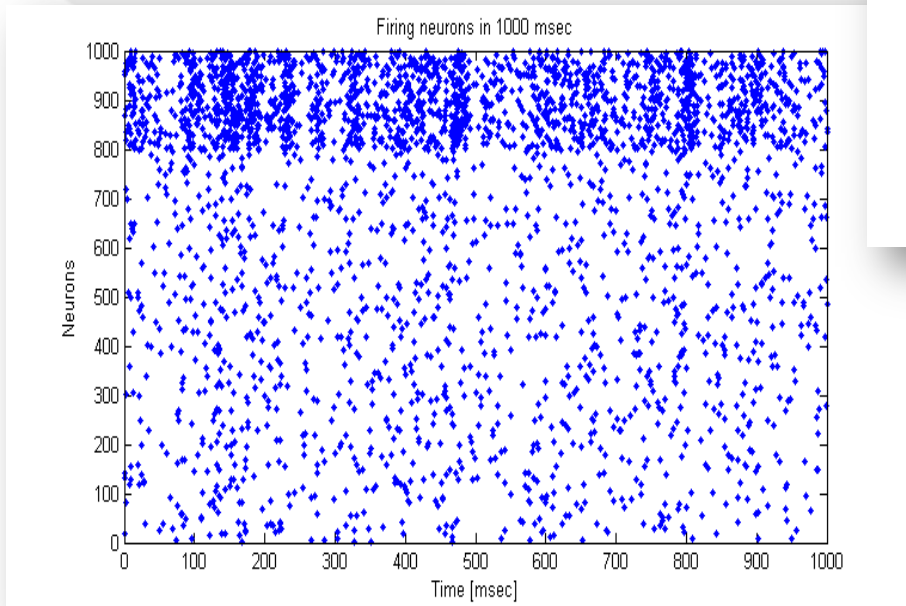
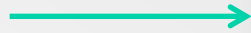
***1000***

***Astrocytes***

***1500***

# Experimental Results

SNAN with 1000  
Neurons and 1500  
Astrocytes



SNN with 1000 neurons

# Experimental results

Comparison, in terms of number of polychronous groups, of the network implementations.

